# RELIABILITY BASED ANALYSIS OF DAM EMBANKMENT, GEOCELLREINFORCED FOUNDATION AND EMBANKMENT WITH STONE COLUMNS USING FINITE ELEMENT METHOD

***A Project Report***

*Submitted by*

NAGENDRA KOLA

*In partial fulfilment of the requirements*
*for the award of the Degree of*

**MASTER OF TECHNOLOGY**

in

**GEOTECHNICAL ENGINEERING**

***Under the guidance of***

Prof. Sarat Kumar Das

**DEPARTMENT OF CIVIL ENGINEERING**
**NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA**
**JUNE 201**

DEPARTMENT OF CIVIL ENGINEERING
NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA
ORISSA, INDIA-769008

# CERTIFICATE

This is to certify that the Thesis Report entitled "**reliability based analysis of dam embankment, geocell reinforced foundation and embankment with stone columns using finite element method** ",submitted by **Mr. *Nagendra kola* Roll no. 212CE1019** in partial fulfilment of the requirements for the award of **Master of Technology in civil Engineering** with specialization in "**geotechnical**" during session 2012-2014 at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any Degree or Diploma.

**Place: Rourkela**

**Date: - 26thMay, 2014**

**Prof. N.ROY**
Dept. of Civil Engineering
National Institute of Technology
Rourkela – 769008

# ACKNOWLEDGEMENT

I would like to express my deepest gratitude to my guide, **Prof.** Sarat Kumar Das for the endless fount of guidance, cooperation and encouragement enabled me to overcome obstacles and complete my project. Their untiring effort and friendly behaviour need special mention.

I also express my sincere gratitude to **Dr. S. K. Sarangi,** Director and **Prof N.Roy,** Head of the Civil Engineering Department, National Institute of Technology, Rourkela, for their advice and providing the necessary facilities for my project work.

I would like to extend my sincere thanks to all the faculty members of the Civil Engineering Department who played a vital role in bringing me to this level.

I am greatly indebted to my parents for their encouragement and endless support that helped me at every step of life. Their sincere blessings and wishes have enabled me to complete my work successfully.

I bow to the Devine power, who led me all through.

**Nagendra kola**

# Table of Contents

## List of Figures

## List of Tables

# ABSTRACT

Geotechnical parameters always associated with uncertainties. The soil properties for a given site disperse within a significant range. Hence, the factor of safety is used in the case of deterministic approach which does not truly account for the uncertainty associated with the soil properties. Also this does not consider the sources and amount of uncertainty associated with the system. This leads to the problem in applying Limit state design of the geotechnical structures. So it is always important to study a given problem in probabilistic manner. In the present study, FORM reliability method was utilized to analysis various geotechnical structures such as dam, geocell reinforced footing and embankment with stone column based on finite element method (FEM). The limit state functions were formulated using response surface methods based on finite element models using commercial software PLAXIS 9.02. Full factorial design is used for development of response surface models. The FORM reliability analysis is performed in all said cases neglecting special variation of soil parameters. The need for reliability analysis and the corresponding factor of safety is discussed. Parametric study has been done by considering the variability in soil parameter.

# INTRODUCTION

**INTRODUCTION:**

Uncertainties in the geotechnical engineering are unavoidable. The geotechnical engineering deals mostly with natural materials. So the variability of the material is inevitable. This is termed as special variability. The soil properties are obtained from field or from laboratory testing and the properties vary depending upon borehole location, number of samples, borehole methods etc. Method of sampling (undisturbed or disturbed), method of laboratory testing, interpretation of statistical results from testing data, the method of analysis for the particular problem such as Meyerhof, Terzaghi, Vesic bearing capacity methods, instrumental error, human error are also considered as uncertainty associated with the performance of the system.

Moreover in some cases though the probability of failure is high but system shows high factor of safety in deterministic analysis. Factor of safety is chosen based on past experience and the outcome of failure. The factor of safety is used in the deterministic approach which account for natural soil variability, measurement errors, statistical approximations, model transformation and limitation in analytical models. This does not consider the sources and amount of uncertainty associated with the system. A factor of safety of 2.5–3.0 is adopted to account this variability in various geotechnical bearing capacity problems. Serviceability of the structure is difficult to estimate using deterministic methods. More over in some cases though the probability of failure is high but system shows high factor of safety in deterministic analysis. Factor of safety is chosen based on past experience and the outcome of failure. The factor of safety is used in the deterministic approach which account for natural soil variability, measurement errors, statistical approximations, model transformation and limitation in analytical models. This does not consider the sources and amount of uncertainty associated with the system. A factor of safety of 2.5–3.0 is adopted to account this variability in various geotechnical bearing capacity problems. Serviceability of the structure is difficult to estimate using deterministic methods.

*Reliability:*

Reliability of the system is the relationship between loads the system must carry and its ability to carry. Reliability of the system is expressed in the form of reliability index (β). This reliability index is related to the probability of failure of the system ( $p_f$ ). Risk and reliability are complementary terms. Risk is unsatisfactory performance or probability of failure. On the other hand reliability is satisfactory performance or probability of success.

Benefits of reliability method in concurrence with conventional design

1. All sources of uncertainties involved in the project are taken into account.

2. Support in decision making regarding risk – cost analysis.

3. Probability of failure can be known for each design methods.

4. The structure can be designed according to serviceability conditions.

5. The overall risk involved in the project is clearly identified.

## SCOPE AND ORGANIZATION OF THE THESIS:

After the brief introduction (Chapter 1), the recent trend in reliability analysis in geotechnical engineering problems is described in Chapter 2. The literature pertaining to reliability analysis, finite element analysis and response surface methods in geotechnical engineering are critically evaluated in this chapter. For the present finite element study commercial software PLAXIS is used.

Chapter 3 describes the use of reliability analysis in stability of reservoir dam embankment under drawn down conditions. Dam drawdown conditions are analyzed using finite element method. Considering the variability in cohesion, angle of internal friction of the soil different FEM models are developed as per full factorial design as per response surface method to formulate the performance function for reliability analysis. The reliability index and probability of failure is calculated using first order reliability method (FORM). While describing the reliability analysis of dam embankments, effect of various soil parameters like cohesion and angle of internal friction on reliability index is also discussed.

Safe bearing capacity of foundation is one of the important stability problems in geotechnical engineering, which depends upon the bearing capacity and the allowable settlement of foundation. In Chapter 4, using PLAXIS, experimental and numerical settlement of footing are compared. The problem has been taken from the T.G.Sitharam & A. Hedge (2013) paper. The settlement is predicted through FE software PLAXIS. The variability in soil properties of the layered soil is discussed. Then after reliability analysis is performed for settlement of Geocell reinforced footing using FEM. Variability in soil parameters are taken into account for reliability analysis. Full factorial design is used in the design of experiments. Response surface model is generated using this input variables and output response. Using this limit state function, reliability Index and probability of failure of the system are calculated.

In chapter 5, stone column embankment is analysed using the FEM. The stability under consolidation process is analysed. The reliability study is carried out for the stability of the embankment with stone columns considering variability in soil parameter. The failure of the embankment is also studied based on reliability analysis.

In Chapter 6, generalized conclusions made from various studies made in this thesis are presented and the scope for the future work is indicated.

# CHAPTER-2

# REVIEW OF LITERATURE

# &

# METHODOLOGY

## 2.1 REVIEW OF LITERATURE:

The reliability analysis in geotechnical engineering developed over the years starting from the probabilistic methods, and some of the studies are discussed as follows. Fardis and Veneziano (1981) developed a probabilistic model based on statistical analysis of liquefaction potential of sands using the results of 192 published cyclic simple shear tests taking into account the uncertainties caused by the effect of sample preparation, effect of system compliance and stress non uniformities. Chowdhury and Grivas (1982) have developed a probabilistic model for progressive failure of slopes. Harr (1987) conducted the extensive study in the application and methods of reliability analysis in civil engineering. Hwang and Lee (1991) considered uncertainties in both site par meters and seismic parameters to calculate probability of liquefaction index, *PL*, based on *SPT N-value* which measures the severity of liquefaction. Low and Tang (1997) have proposed the procedure to calculate the Hasofer Lind second moment reliability index using spread sheet. Low (2003) explained the practical probabilistic slope study with case studies. Low (2005) compared the expanding ellipsoid, Hasofer-Lind method and FORM. Low (2005) analyzed the retaining walls for overturning and sliding. Correlated normal variables have used in the study. Monte Carlo simulation method is a probabilistic method which uses random number generators. Greco (1996) and Malkawi et al (2001) have analyzed the slopes using Monte Carlo simulation methods. But it involves high computational expenses. Phoon and Kulawy (1999) have explained the variation in geotechnical property. He explained about measurement error, transformation uncertainty and soil variability. Coefficient of variation has been evidently explained by him. Babu et al. (2007) have analysed the stability of earthen dams by Monte Carlo simulations and conducted the reliability analysis. Babu and Srivastava (2010) have conducted the reliability study on earth dams by developing response surface models by Finite difference method. Babu and Basha (2008) have analyzed the sheet pile walls by target reliability approach. Inverse first order reliability method has used to analyze the anchored cantilever sheet pile wall. Christian et al. 1994, Low 2003, Low and Tang 1997 have proposed reliability-based approaches to slope stability problems. Xue and Cavin (2007) considered the variables in polar coordinates and the reliability index defined with the Hasofer-Lind method is formulated as a function of the soil properties and the slip surface. With genetic algorithm, the nonlinear programming problem has solved. In this method, the reliability index and critical slip surface are found concurrently.

## 2.2 METHODOLOGY:

The parameters involved in the particular problem are studied. The random variables are chosen which affect the required output. The variability of the random variables is inspected. Then using Full Factorial design, experimental design is developed. For each set of input variables required output is developed using Finite Element Method. These set of input variables and its corresponding output is used to develop the linear response models. These linear response models are used to develop the limit state function. First order reliability method is used to find out the reliability index. The reliability index is minimized using Excel solver with the constraint as performance function. From this reliability index probability of failure is obtained. The flow chart for the above considered for the present study is presented in Figure 2.1 as shown below.

`

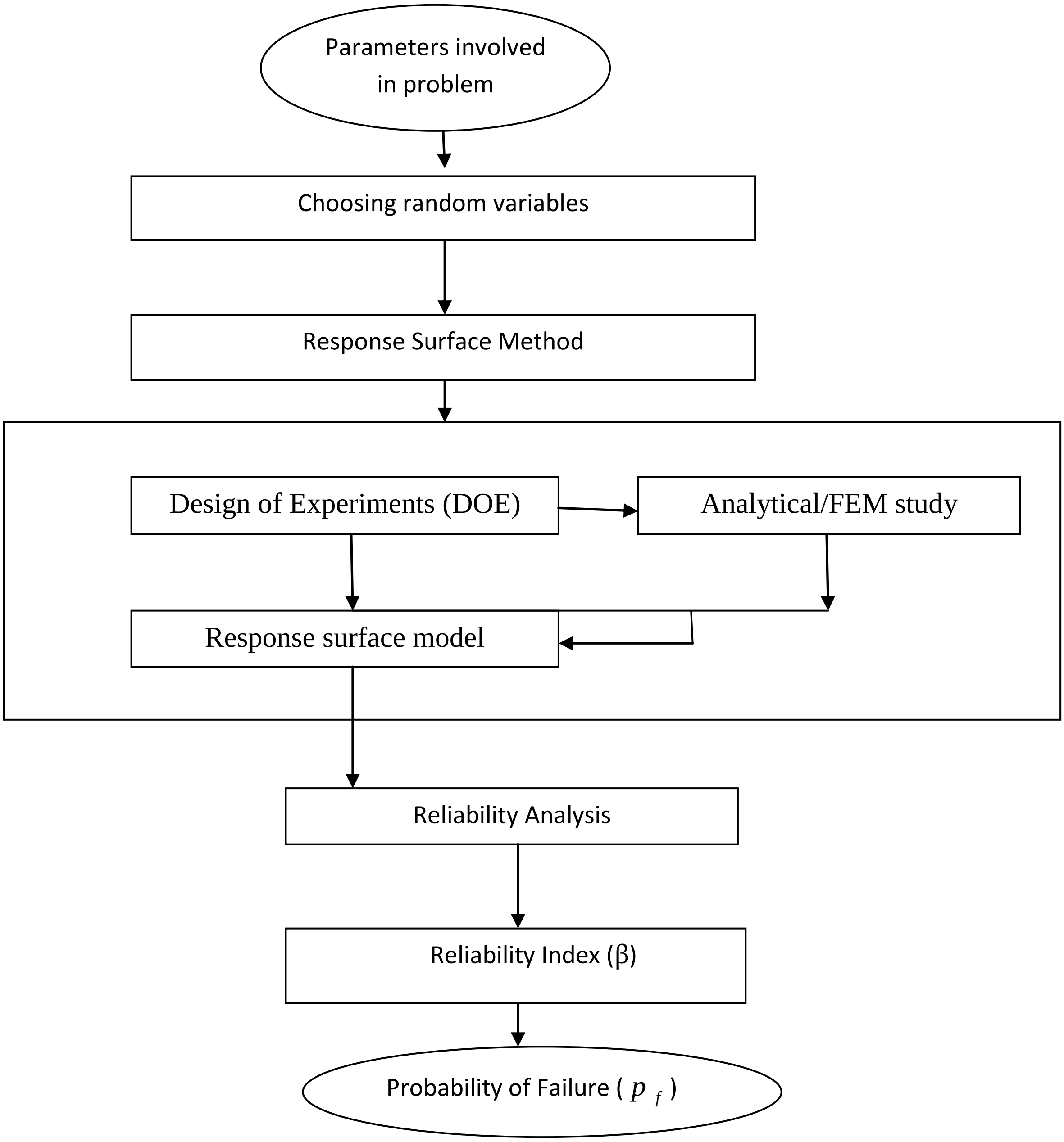


Fig: 2.1 flow chart for the reliability analysis

### 2.2.1 Finite Element Method:

It allows modelling complicated non linear soil behaviour through constitutive model, various geometrics with different boundary conditions & interfaces. It can predict the stresses, deformations and pore pressures of a specified soil profile.

**PLAXIS*:***

According to Burd (1999), the initiation of this Finite Element Program was held at Delft University of Technology Netherland by Pieter Vermeer in 1974. PLAXIS name was derived from Plasticity AxiSymmetry, a computer program developed to solve the cone pentrometer problem by Pieter Vermeer and De borst. The commercial version of PLAXIS was released in 1987. Earlier version of PLAXIS was in DOS interface. PLAXIS V-7 was released in windows with automated mesh generation. Advanced soil models were also incorporated.

***2D Finite Element Model in PLAXIS:***

Axisymmetric and Plane strain conditions with two translation degrees of freedom along x-axis and y-axis are available in PLAXIS. However, axisymmetric models are applied only for circular structures with a uniform radial cross section. The loads are also assumed as circular symmetric around the central axis. In the plane strain model the displacements and strains in z-direction are assumed to be zero. But normal stresses in z-direction are considered.

***Elements***

PLAXIS 2D uses 2nd order 6-node with 3 gauss point & 4th order 15-node with 12 gauss point triangular elements to model the soil. 3 node & 5 node beam elements are available to model shell, retaining wall and other slender members. 3-node element has 2 pair of Gaussian stress points and 5-node element has 4 pair of Gaussian stress points. Bending moments and axial forces of these Plates are calculated from the stresses at the Gaussian stress points.

***Constitutive models***

*Mohr-Coulomb Model*

This is the simple model to represent the soil behaviour. This is an elastic perfectly plastic soil model. The model engages with five parameters: Cohesion (c), Angle of Friction (ϕ), Dilatancy angle (Ψ), Young's modulus (E) and Poisson's ratio (ν).

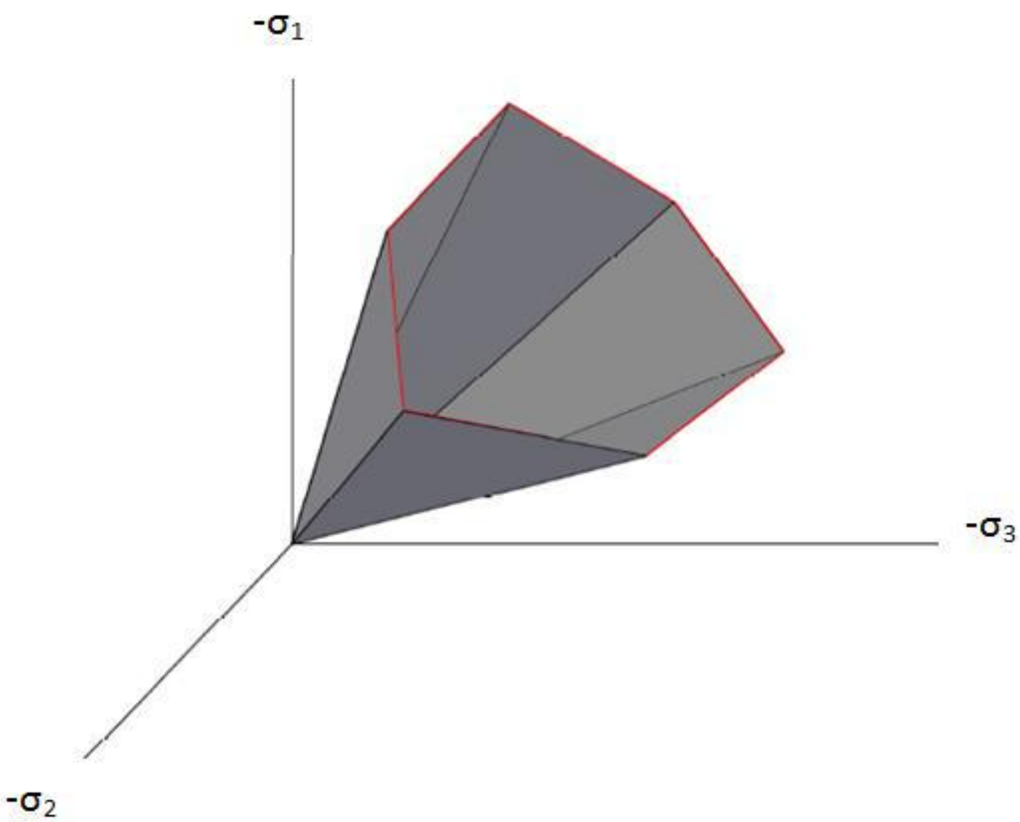


Fig: 2.2 Yielding surface at principle stress space (c=0) for M-C model

*Linear Elastic Model*

This model is based on Hooke's law. The model involves with two parameters: Young's modulus (E) and Poisson's ratio (ν). The model is used to simulate the structural elements in soil such as footing, Pile or Rock.

***Mesh Properties***

PLAXIS involves automatic mesh generation. PLAXIS produces unstructured mesh generation. The mesh generation is based on robust triangulation procedure. Global refinement (to increase the number of elements globally), Local refinement (to increase the number of elements in particular cluster), Line refinement (to increase the element numbers at the cluster boundaries), Point refinement (increasing the element coarseness around the point) are available to obtain the better results. The number of mesh elements considerably affects the results. So sensitivity study on mesh elements for each analysis should be investigated.

***Model Simulation:***

In the present study PLAXIS 9.0 is used to simulate the Settlement of footing, Slope stability and Retaining wall.

***Strength reduction technique:***

Sudden increase in the dimensionless displacement of soil mass and the algorithm unable to converge within the iteration limit can be considered as the failure of the slope. In PLAXIS arc length procedure provides the strength displacement curves. Arc length control composes the procedure strong since the procedure need not be associated with a non-converging iterative procedure. The method avoids the strength parameters decreases beyond critical value. When further reduction in the shear strength parameters is not possible, the construction has collapsed and at that point the safety factor is obtained. During the calculation phase, Arc length control should be activated. In the calculation of Factor of safety Young's modulus (E) of the soil has no influence and Poisson's ratio (ν) has negligible influence.

$$FOS = \frac{Available\, shear\, strength}{Shear\, strength\, at\,\, failure}$$

In PLAXIS this FOS is indicated in terms of sum of incremental multiplier (ΣMsf). The displacement of the soil during failure has no practical meaning. When the Phi-c reduction method is applied to advanced soil models, it follows Mohr-Coulomb failure criteria.

In the slope Stability analysis, initial stresses are developed in the calculation stage according to gravity loading method because this always results in equilibrium stress state. But K0 procedure does not applicable for sloping ground. During the gravity loading self weight of the soil and generated pore pressure are activated. If the gravity loading is used, it causes displacements. So in the next calculation phase the displacement should be reset to zero. The initial stress condition in K0 procedure is generated by Jaky's formula (Jaky 1944)

$$k_0 = 1 - \sin\varphi'$$

$\varphi'$ = effective friction angle.

***Convergence criteria:***

Convergence study is conducted for mesh coarseness. By increasing the number of elements the variation in the output parameter is inspected. The number of mesh element is varied and inspected until the output parameter for the two successive meshing is negligible. If the system fails before it reaches the maximum number of step then the calculation is controlled by allowing tolerated error. The mesh size can be inspected if it does not converge in the calculation stage.

## 2.2.2 Response Surface Method

The response surface method (RSM) originated by Box and Wilson (1951) is a collection of statistical and mathematical techniques helpful for developing, improving and optimizing processes through empirical model building. Response surface methodology is the practice of adjusting predictor variables to move the response in a desired direction to an optimum by iteration. The method generally engages a combination of both computation and visualization. The use of quadratic response surface models makes the method simpler than standard nonlinear techniques for determining optimal designs. The Response surface method consists of design of experiments and response surface analysis. Response surface models are multivariate polynomial models. They typically arise in the design of experiments, where they are used to determine a set of design variables that optimize a response.

In a designed experiment, the data-generating process is manipulated to improve the quality of information and to eliminate unused data. An experiment is a series of tests, called runs, in which changes are made in the input variables in order to identify the causes for changes in the output response. A common goal of all experimental designs is to collect data as cheaply as possible while providing sufficient information to precisely estimate model parameters.

Response surface analysis aims to interpolate the available data in order to predict the correlation locally or globally between variables and objectives. If the data follows a flat surface, a first order model is usually sufficient.

A simple model of a response y in an experiment with two controlled factors $x_1$ and $x_2$ look like this:

$$y=\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_{12} x_1 x_2+\varepsilon$$

The response can be characterized graphically, either in the three-dimensional space or as contour plots that aid visualize the shape of the response surface.

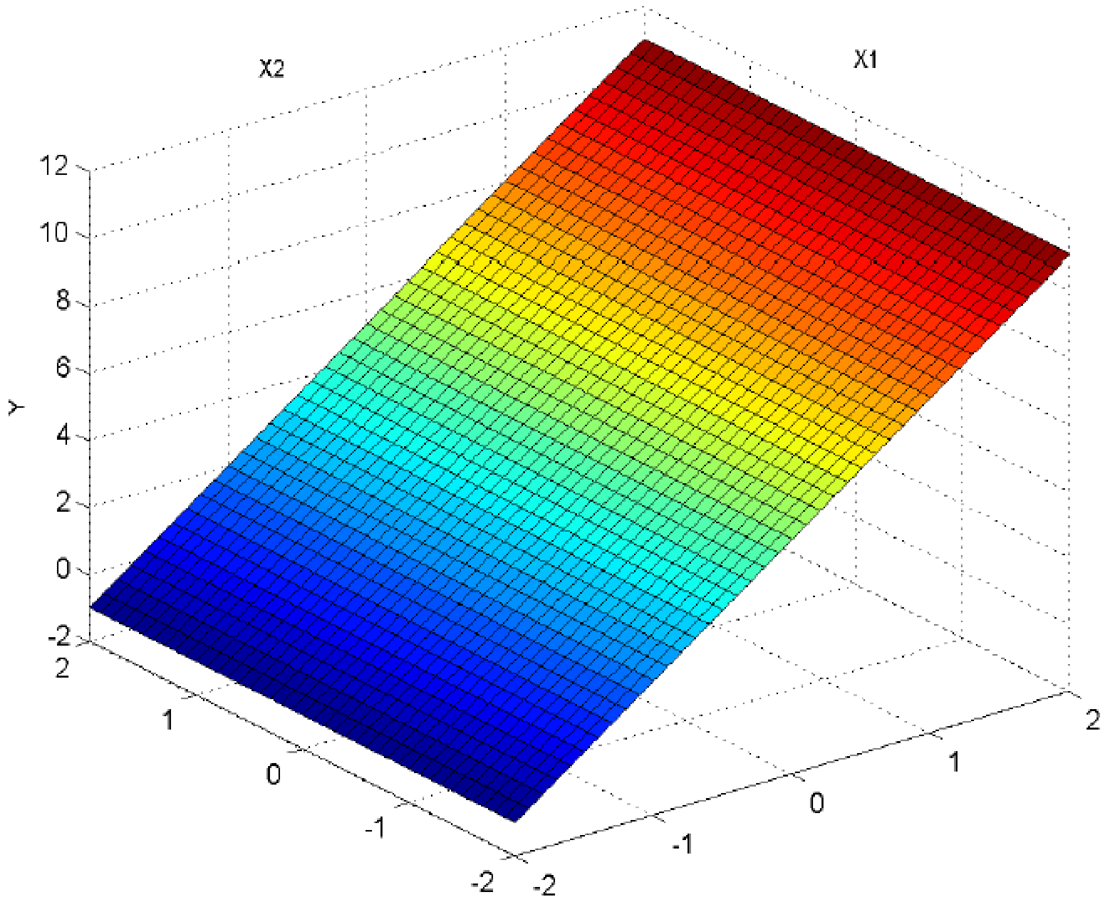


Fig 2.3 Linear Response surface

Here $\varepsilon$ includes both experimental error and the effects of any uncontrolled factors in the experiment. The terms $\beta_1 x_1$ and $\beta_2 x_2$ are main effects and the term $\beta_{12} x_1 x_2$ is a two-way interaction effect. A designed experiment would systematically manipulate and while measuring y, with the objective of accurately estimating $\beta_0$, $\beta_1$ , $\beta_2$ , and $\beta_{12}$.

If there is curvature in the data, a first order model would show a significant lack of fit. A higher order model must be used to "mold" to the curvature. Polynomial models are generalized to any number of predictor variables $x_i$ (i = 1, N) as follows:

$$y=\beta_0+\sum_{j=1}^{k}\beta_j x_j+\sum_{j=1}^{k}\beta_{jj}x_j^2+\sum_{i<j}\sum_{=2}^{k}\beta_{ij}\,x_i\,x_j+\varepsilon$$

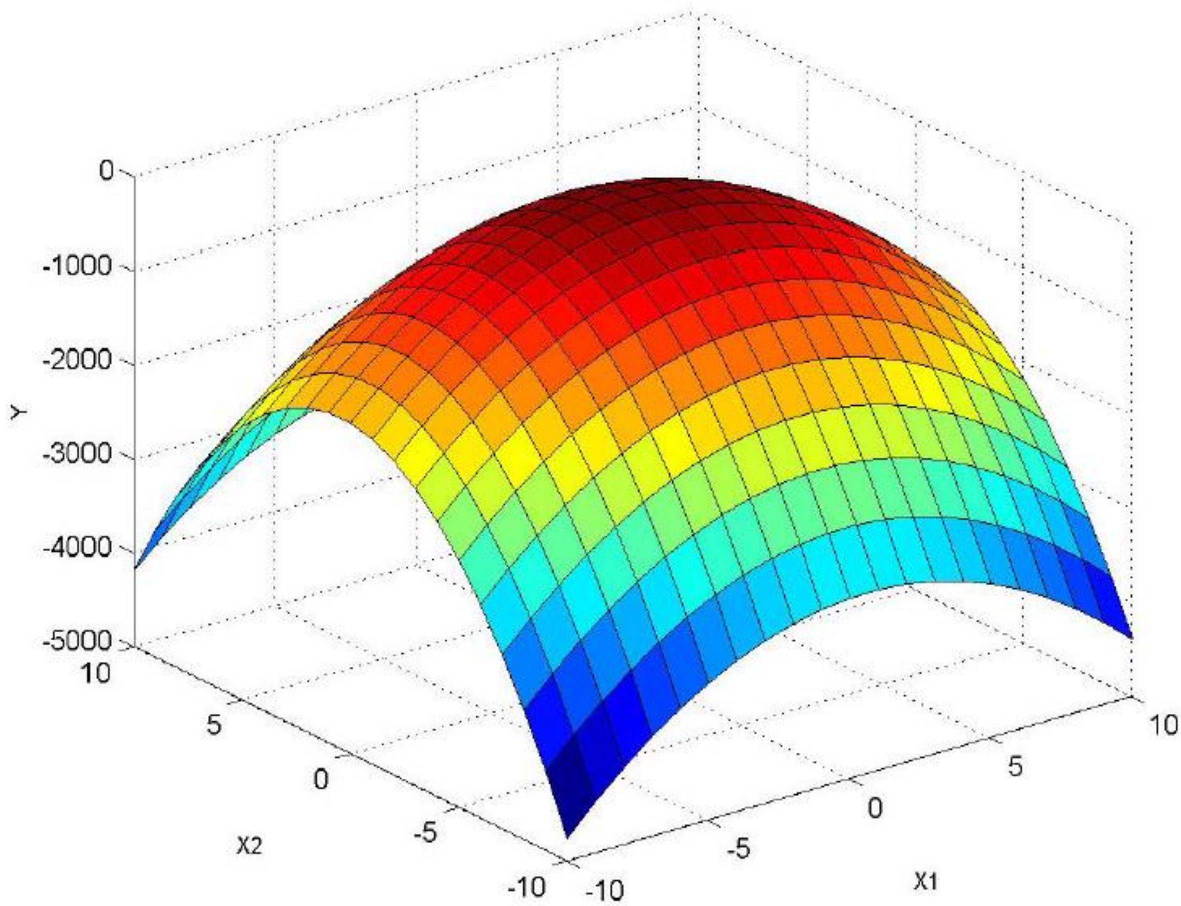


Fig 2.4 Non-linear response surface

## *Design of Experiments: (DOE)*

*Factorial Designs*

A factorial experiment is an experimental tactic in which design variables are varied together, instead of one at a time. In experiments, factorial designs are used to investigate the joint effects of the factors on a response variable. The important special case of the factorial design is two level factors in which each of the k factors of interest has only two levels. In this, each design has $2^k$ experimental trials. These designs are known as $2^k$ factorial designs. The $2^k$ design is the basic building block. So this is used to create other response surface designs. A $2^k$ design is useful at the start of a response surface study. Screening experiments should be performed to identify the important system variables. This design is also used to fit the first order response surface model.

*Two-level full factorial design:*

$2^k$ Factorial design:

The simplest design in $2^k$ series is with two factors $x_1$ and $x_2$ and this run in two levels.

Mat lab code for design of experiments:

dFF2 = ff2n (n)

dFF2 is R-by-C, where R is the number of treatments in the full-factorial design. Each row of dFF2 corresponds to a single treatment. Each column contains the settings for a single factor, with values of 0 and 1 for the two levels.

If the number of parameters involved in the design is 3, then the design can be generated in Mat lab as follows. These binary set don’t have any meaning and simply considered as design set.

>> dFF2 = ff2n (3)

dFF2 =

0 0 0

0 0 1

0 1 0

0 1 1

1 0 0

1 0 1

1 1 0

1 1 1

In this experimental design eight set of data has generated for 3 input parameter. 0 and 1 are then estimated as $\mu+1.65\sigma$ and $\mu-1.65\sigma$ . μ is the mean of the variable. σ is standard deviation of the corresponding variable.

σ =μ * cov

Cov is the coefficient of variation of the particular parameter of the soil. The decoded design sets (*x1, x2, and x3*) are used to conduct experiments and output response *(y1)* is obtained. Using this eight set of input-output parameters linear or nonlinear regression model is developed using MS Excel.

### 2.2.3 Reliability Analysis:

Reliability is the property that the structure will not attain specified limit state during specified time.

**Terminology:**

Mean:

Mean is an average value of data set. It is used to measure the central tendency of data. It is also known as 1st central moment.

Coefficient of variation :( COV)

COV is a statistical measure of the dispersion data around the mean. Higher the COV higher the dispersion about mean.

Covariance:

Degree of linear relationship between two random variables(x, y) indicated by covariance.

$$Cov\ (x, y) = E\ [(x-\mu_x)\ (Y-\mu_y)] = E\ [XY - \mu_X\ \mu_Y] = E\ (XY)-E(X)\ E(Y)$$

Correlation coefficient:

It is the ratio of the covariance of two random variables to the product of standard deviation of individual variables ($\sigma_x$, $\sigma_y$)

$$P_{xy} = \frac{Cov\ (x,y)}{\sigma x \sigma y}$$

$$-1 \leq P_{xy} \leq +1$$

It is a non dimensional parameter.

Continuous random variables:

To quantify the uncertainties in random variables the mathematical model witch satisfying the probability density function, cumulative distribution function, and probability mass function is used. The continuous random variables may follow normal distribution, β-distribution or non normal distribution.

Properties of normal distribution:

1, the parameters varies between $-\infty \leq x \leq \infty$

2, Mode, mean, and median values are same

3, it is exactly symmetric about mean

$$f_x(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left[ -\frac{1}{2} \left( \frac{x - \mu_x}{\sigma_x} \right)^2 \right] \ , \ -\infty \leq x \leq \infty$$

Reliability methods:

A resistance- load model of a structural component is considered as example to understanding the concept of reliability, the failure occurs when the load (Q) on the structure exceeds the resistance of the structure. The load is used to indicate any structure that have tends to fail, while resistance(R) indicates any structure that resists failure.

$f_q$ = probability density function of load

$f_r$ = probability density function of resistance.

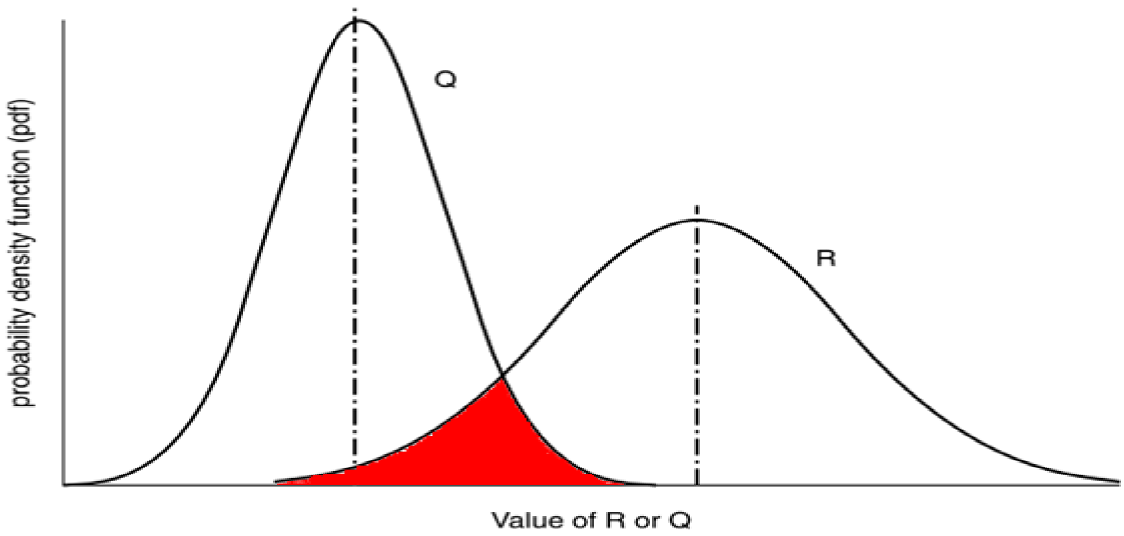


Fig 2.5: graphical representation of load and resistance interface region

The reliability is defined as the probability that the load(q) did not exceed the resistance(r) as follows:

Reliability = $p(r > q) = p(r - q > 0)$

The red region in figure indicates the probability of failure and its converse denotes the probability of safety (reliability). The probability of failure magnitude is function of degree of overlap of the two distributions. The higher the shaded area, the higher is the probability of failure.

The probability of failure ($p_f$) can mathematically expressed as

$$p_f = \iint_{\Omega} f_q(q) f_r(r) dr ds$$

$$\text{Reliability, } R_0 = 1 - \iint_{\Omega} f_q(q) f_r(r) dr ds$$

It is the simple case where load and resistance are only two random variables involved. But in geotechnical problems the load and resistance are the functions of several random variables.

The failure domain ($\Omega$ ?) always need not to be an analytical expression problem become more complicated has number of random variables increases. Then other methods like simulation based reliability methods or the analytical reliability approximation methods ($1^{st}$ and $2^{nd}$ order reliability methods) should have employed.

The international standards organization divided the design procedures into 3 groups depending up on complexity involved in the probability theory.

Level 1: semi probabilistic method:

In level 1 reliability methods each uncertain parameter carries one characteristic value only .load and resistance factor design is the example.

Level 2: approximate probabilistic method:

In level 2 reliability methods each uncertain parameter carries two values (mean & variance), and also involves correlation between the parameters. The first order second moment method (FOSM) is the one of the example of first order second moment method. it uses mean and coefficient of variation. The first two moments and approximated by Taylor series of expansion.

Level 3: fully probabilistic method:

Depend up on the probability distribution of random variable. Probability of failure has to be calculated then joint distribution of all uncertain parameters .so it very complex.

Limit state function:

The load and resistance are related by derive a mathematical model is limit sate function. The limit state equation is represented as

$$Z = (R-Q) = g(X_1, X_2, X_3, \ldots\ldots\ldots\ldots X_N)$$

Z = Safety margin.

If the limit state function is equals to 0 called limit sate equation or failure surface equation

$$g(X_1, X_2, X_3, \ldots\ldots\ldots\ldots X_N) = 0$$

$$\text{Reliability } R_0 = 1 - p_f$$

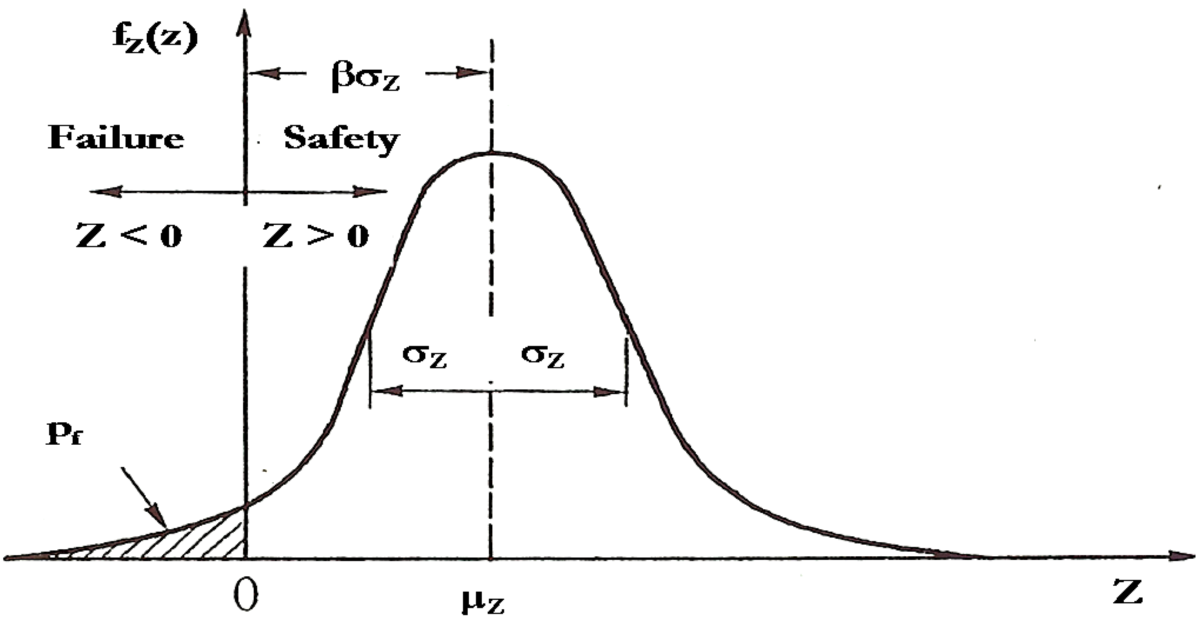


Fig 2.6: safety margin distribution ( $Z = R - Q$ )

Cornell reliability index,

$$\beta = \frac{\mu_z}{\sigma_z}$$

And

$$p_f = \Phi(-\beta)$$

$\mu_z$ = mean of the random variable Z

$\sigma_z$ = standard deviation of the random variable Z

Φ = cumulative distribution function

First order reliability methods (FORM):

The performance functions mean and variance can be estimated by using Taylor series expansions first terms. This method is known as first order second moment method because variance is form of second moment.

FOSM:

Q is the loading on the system R is the available resistance of that system. R and Q are uncertainties. R and Q have the mean and expected values of variance and covariance. The margin of safety is explained by the performance function of the system.

The limit state function equation can be written as

$M = R - Q = 0$

Probability of failure is

$$p_f = p[(R - Q) \leq 0]$$

Cornell reliability index β is calculated as

$$\beta = \frac{(\mu_R - \mu_Q)}{\sqrt{\sigma_R^2 - \sigma_Q^2}}$$

M = linear function of variables

$$M = b_0 + b_1 X_1 + b_2 X_2 + .............. + b_n X_n$$

$$\mu_M = B_0 + \sum_{I=1}^{n} b_i \mu_i$$

$$\sigma_M^2 = \sum_{i=1}^{n} b_i^2 \sigma_i^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \rho_{ij} b_i b_j \sigma_i \sigma_j$$

If the variables not correlated

$$\sigma_M^2 = \sum b_i^2 \sigma_i^2$$

The mean value 1st order 2$^{nd}$ moment method (MVFOSM) is used for variables with non linear functions.

$$\mu_M = g(\mu_1, \mu_2, \mu_3 .................. \mu_n)$$

$$\sigma_M^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \left( \frac{\partial g}{\partial X_i} \right)$$

In this method, the failure surface is linear zed at the mean value in the formation of limit state function and hence an unacceptable error values occur on approximating non linear failure surface.

**Hasofer – Lind reliability method (Advanced first order second moment method)**:

FOSM method develops by Hasofer and Lind in 1974 depending up on geometry. It is useful in calculation of first order approximation of the probability of failure. Hansofer - Lind reliability index is applicable for the normally distributed variables. If the non normal variables are there it is compulsory to transfer into normal variable. For all non normal random variables the equivalent normal mean and standard deviation approximately calculated at the design point. The reliability index is calculated by transform original coordinate system into reduced coordinate system. The distance between the peaks of the multivariate distribution of the input variables to the failure surface defined by limit state function in dimensionless space. The Hasofer – Lind method is also called as FORM. Lack of invariance can be overcome by this method.

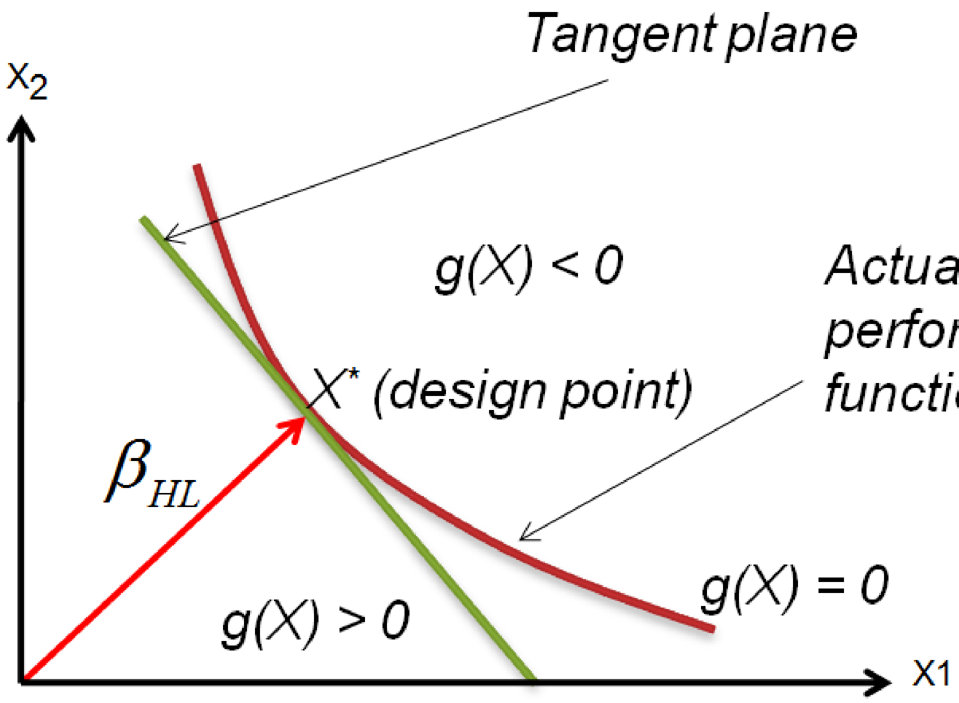


Fig 2.7: Hasofer- Lind reliability index for non liner performance function

Basic variables are uncorrelated and normal in this method. Transfer the basic variables into standard normal variables (with μ=0, $\sigma = 0$).

$$x_i = \frac{X_i - \mu_i}{\sigma_i},\ i = 1,2,.............n$$

Reduced space coordinates limit state equation

$$g_1(x_1, x_2, .........., x_n) = 0$$

Take X* on g(x) = 0 as design point

The most probable point (MPP) of failure is X*. So the minimum distance between X* and origin is called as reliability index. The minimum distance computation acts as constrained optimization problem for non liner limit state function.

$$\beta_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$p_f = \varphi(-\beta_{HL})$$

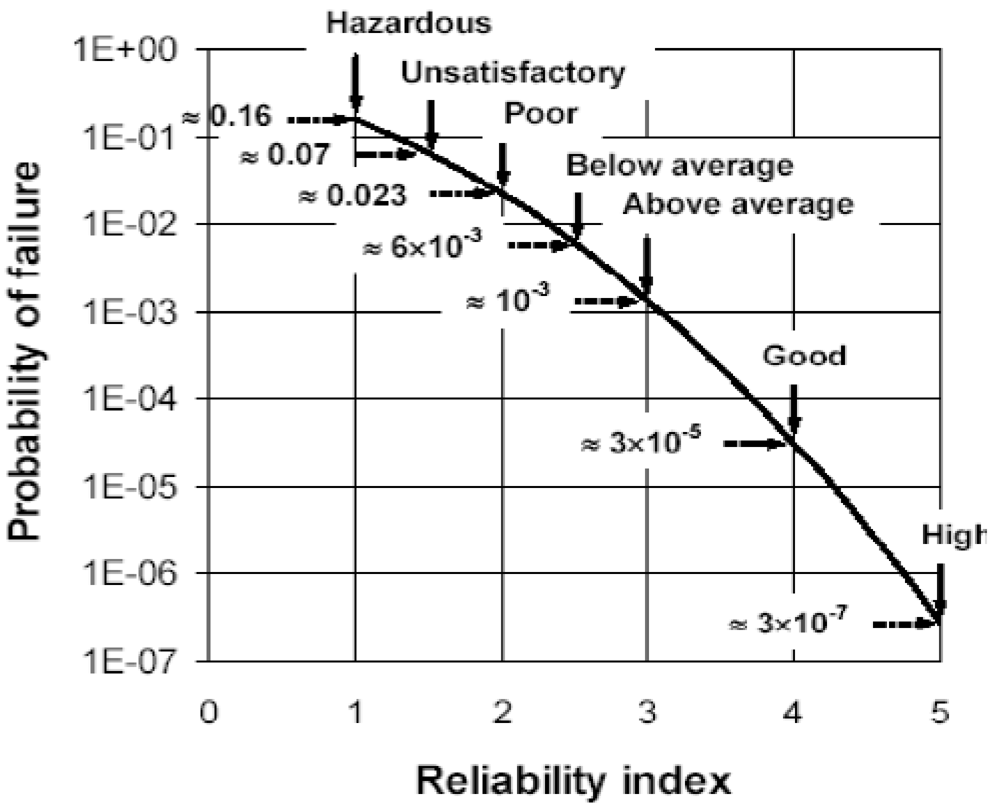


Fig 2.8: Graph between probability of failure and reliability index USACE (1997)

CHAPTER-3

# RELIABILITY ANALYSIS OF STABILITY OF RESERVOIR DAM EMBANKMENT UNDER DRAWN DOWN CONDITIONS

# 3. RELIABILITY ANALYSIS OF STABILITY OF RESERVOIR DAM EMBANKMENT UNDER DRAWN DOWN CONDITIONS

## 3.1 INTRODUCTION:

The fast draw down of reservoir water level May causes the instability of the dam due to higher pore water pressure developed in the dam. There are various analytical and graphical methods to stability analysis of the dam under these conditions. But there are many assumptions and uncertainties involved in the collection of soil parameters and the traditional formulas used so that calculated factor of safety may not my be reliable .The stability analysis of such draw down conditions can be down by reliability analysis in this chapter. To analysis such a conditions the FEM (finite element method) with transient ground flow calculation is used. The stability analysis was done by using transferred pore pressure obtained from the ground water flow analysis to deformation analysis. The $2^k$ design factor is used to design the experiments the response surface model is used to develop the mathematical model .the FEM package PLAXIS (Plax flow) is used for the transient flow analysis and the cohesion of core ($C_c$), angle of internal friction of core material ($ø_c$), angle of share resistance of dam fill ($ø_f$) are the random variables used in the reliability analysis.

## 3.2 ANALYSIS OF RESERVOIR DAM EMBANKMENT:

The figure 3.1 shows the geometrical features of the dam embankment. The figure 3.2 shows the PLAXIS modelling of the dam and the figure 3.3 shows the displacement in the soil as deformed mesh. The figure 3.4 shows the critical failure surface

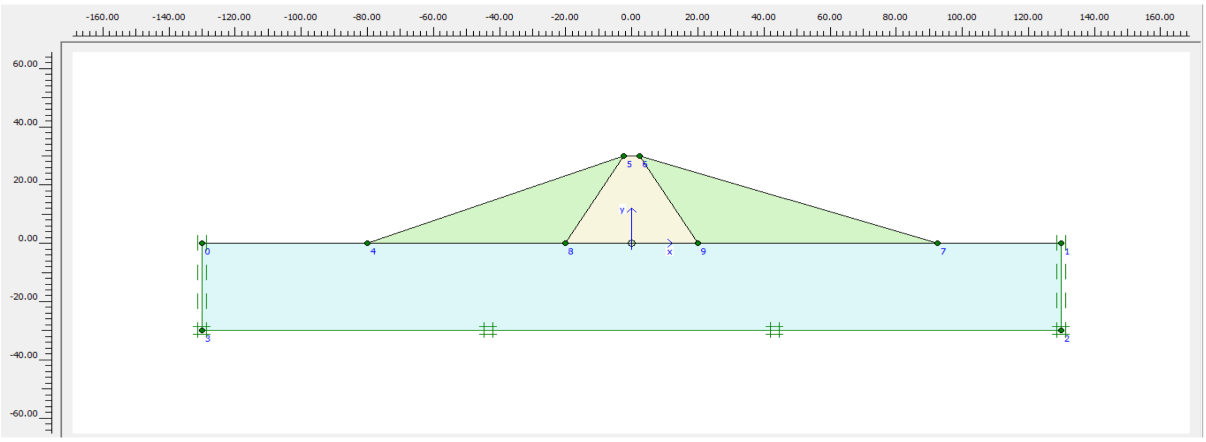

Fig 3.1: geometrical model of dam embankment

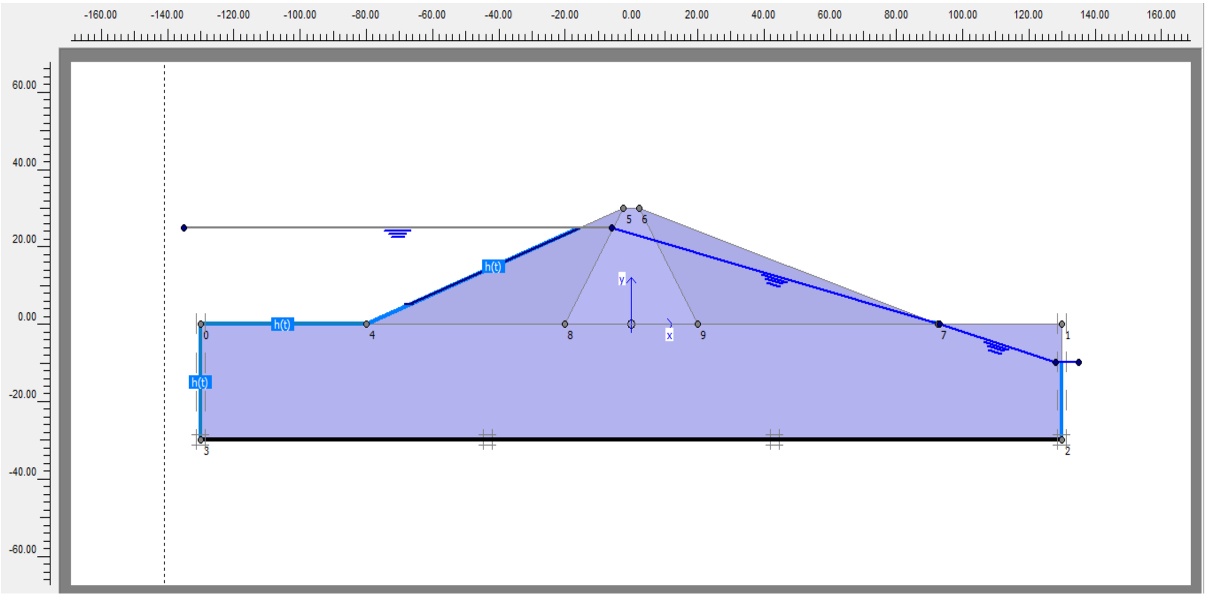

Fig3.2: PLAXIS modelling

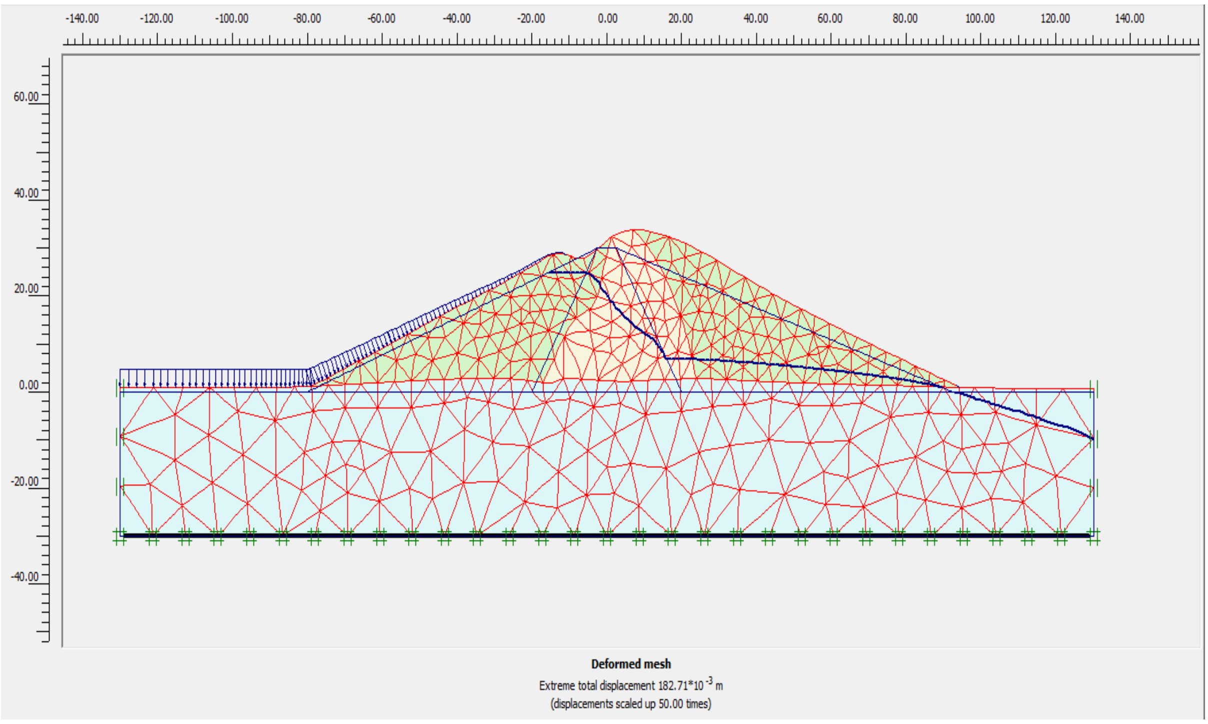


Fig3.3: deformed mesh

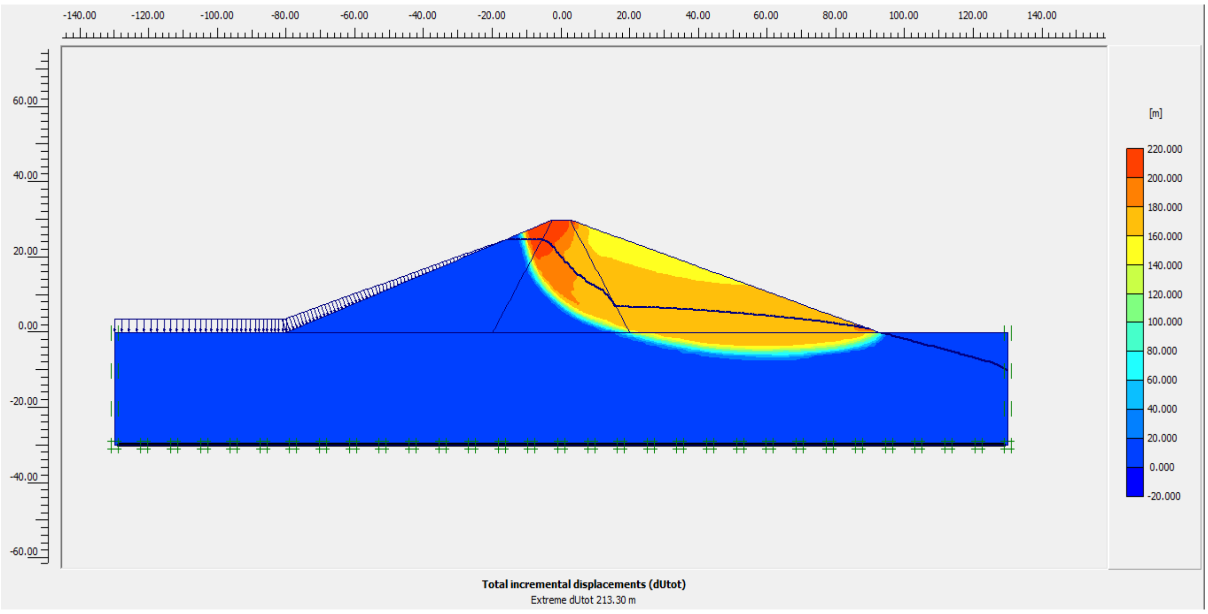


Fig3.4: critical failure surface

### 3.3 RELIABILITY ANALYSIS OF THE DAM EMBANKMENT:

The uncertainties involved in the special variation of soil are considered. The strength criteria parameters of the soil are taken as random variables.

Table: 3.1 COV and Mean of soil parameters

| | Mean(μ) | |
|---|---|---|
| | C | $ø^0$ |
| Core | 10 | 24 |
| fill | 0 | 30 |
| COV (%) | 20 | 13% |

The regression analysis performed by least square error approach. The reliability index is calculated based on this mathematical model developed from response surface model.

The parameters are 1) uncorrelated normally distributed

2) correlated normally distributed

Quantify the each point in the design set by considering the $\mu+1.65\sigma$ (lower limit) and ($\mu-1.65\sigma$)(upper limit) of the normally distribution of the parameters.

Full factorial design:

By Mat lab code

```
>>ff2n (3)

     0     0     0
     0     0     1
     0     1     0
     0     1     1
     1     0     0
     1     0     1
     1     1     0
     1     1     1
```

Table: 3.2 Factor of safety (Fs) of dams of 8 sampling points in response surface model by PLAXIS:

| | Core | Core | FILL | |
|---|---|---|---|---|
| | $C_C$ | $C_f$ | $Ø_f$ | $F_s$ |
| μ+1.65σ | 13.3 | 29.148 | 36.435 | |
| μ-1.65σ | 6.7 | 18.852 | 23.565 | |
| | 13.3 | 29.148 | 36.435 | 1.973 |
| | 13.3 | 29.148 | 23.565 | 1.182 |
| | 13.3 | 18.852 | 36.435 | 1.952 |
| | 13.3 | 18.852 | 23.565 | 1.185 |
| | 6.7 | 29.148 | 36.435 | 1.954 |
| | 6.7 | 29.148 | 23.565 | 1.181 |
| | 6.7 | 18.852 | 36.435 | 1.949 |
| | 6.7 | 18.852 | 23.565 | 1.184 |

The linear surface model developed by regression analysis is

$$F_s = -0.2549+0.000909* C_C +0.0004856* C_f + 0.06013986* Ø_f$$

$$(R^2=0.9996;\ R^2_{adj}=0.9997)$$

**Case: 1** (uncorrelated normally distributed)

The developed performance function

$$G(x) = F_s -1$$

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$X = \frac{x-\mu}{\sigma}$$

$X'$ = matrix of x values

At starting x is taken as mean value of the parameter.

The min distance between the origin and the deign point is reliability index obtained using spared sheet calculation

$\beta = 2.43$

The failure probability of slope

$P_f = \Phi\ (-\beta) = 0.007547$

Case2: Correlated normally distributed parameters(c, ø)

The parameters c and ø are linearly correlated with -0.25 coefficient of correlation

C= Correlation matrix

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$\beta = 2.434$

$P_f = \Phi(-\beta) = 0.007449$

The parameters c and ø are linearly correlated with -0.50 coefficient of correlation

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$\beta = 2.4396$

$P_f = \Phi(-\beta) = 0.007351$

Case3: the parameters are un correlated log normal distributed

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$\beta = 2.8662$

$P_f = \Phi(-\beta) = 0.002077$

Case4: the parameters are correlated log normal distributed

a, coefficient of correlation= -0.25

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

Min $B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$

$\beta = 2.8748$

$P_f = \Phi(-\beta) = 0.002021$

b, coefficient of correlation= -0.5

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| | -0.5 | 1 | 0 |

| | | | |
|---|---|---|---|
| $C_f$ | | | |
| $Ø_f$ | -0.5 | 0 | 1 |

Min $B_{HL} = \underset{G(Z^*)=0}{min} \sqrt{(X')^t (X')}$

β = 2.8848

$P_f = \Phi(-\beta) = 0.001958$

Table: 3.3 Factor of safety (Fs) of dams of 8 sampling points in response surface model by PLAXIS: steady state analysis

| | Core | Core | FILL | |
|---|---|---|---|---|
| | $C_C$ | $C_f$ | $Ø_f$ | $F_s$ |
| μ+1.65σ | 13.3 | 29.148 | 36.435 | |
| μ-1.65σ | 6.7 | 18.852 | 23.565 | |
| | 13.3 | 29.148 | 36.435 | 1.781 |
| | 13.3 | 29.148 | 23.565 | 1.196 |
| | 13.3 | 18.852 | 36.435 | 1.813 |
| | 13.3 | 18.852 | 23.565 | 1.189 |
| | 6.7 | 29.148 | 36.435 | 1.978 |
| | 6.7 | 29.148 | 23.565 | 1.19 |
| | 6.7 | 18.852 | 36.435 | 1.782 |
| | 6.7 | 18.852 | 23.565 | 1.325 |

The linear surface model developed by regression analysis is

$$F_s = 0.19828-0.0112121* C_C +0.0008741* C_f + 0.047668* Ø_f$$

$$(R^2=0.963; R^2_{adj}=0.9614)$$

**Case: 1** (UN correlated normally distributed)

The developed performance function

$$G(x) = F_s -1$$

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t(X')}$$

$$X = \frac{x-\mu}{\sigma}$$

X' = matrix of x values

At starting x is taken as mean value of the parameter.

The min distance between the origin and the deign point is reliability index obtained using spared sheet calculation

$$\beta = 2.839$$

The failure probability of slope

$P_f = \Phi\ (-\beta) =0.00226$

Case2: Correlated normally distributed parameters(c,ø)

The parameters c and ø are linearly correlated with -0.25 coefficient of correlation

C= Correlation matrix

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 2.7574

$P_f = \Phi(-\beta) = 0.002913$

The parameters c and ø are linearly correlated with -0.50 coefficient of correlation

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$\beta = 2.682239$

$P_f = \Phi(-\beta) = 0.003657$

Case3: the parameters are uncorrelated log normal distributed

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$\beta = 3.4747$

$P_f = \Phi(-\beta) = 0.000256$

Case4: the parameters are correlated log normal distributed

a, coefficient of correlation= -0.25

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| | -0.25 | 1 | 0 |

| | | | |
|---|---|---|---|
| $C_f$ | | | |
| $Ø_f$ | -0.25 | 0 | 1 |

Min $B_{HL}$ = $\underset{G(Z^*)=0}{min}\sqrt{(X')^t(X')}$

β = 3.2975

$P_f$ = Φ (-β) =0.000488

b, coefficient of correlation= -0.5

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

Min $B_{HL}$ = $\underset{G(Z^*)=0}{min}\sqrt{(X')^t(X')}$

β = 3.1315  $P_f$ = Φ (-β) =0.000864

Table: 3.4 Factor of safety (Fs) of dams of 8 sampling points in response surface model by PLAXIS: Rapid draw down

| | Core | Core | FILL | $F_s$ |
|---|---|---|---|---|
| | $C_C$ | $C_f$ | $Ø_f$ | |
| μ+1.65σ | 13.3 | 29.148 | 36.435 | |
| μ-1.65σ | 6.7 | 18.852 | 23.565 | |
| | 13.3 | 29.148 | 36.435 | 0.817 |
| | 13.3 | 29.148 | 23.565 | 0.54 |
| | 13.3 | 18.852 | 36.435 | 0.833 |
| | 13.3 | 18.852 | 23.565 | 0.546 |
| | 6.7 | 29.148 | 36.435 | 0.81 |
| | 6.7 | 29.148 | 23.565 | 0.538 |
| | 6.7 | 18.852 | 36.435 | 0.826 |
| | 6.7 | 18.852 | 23.565 | 0.541 |

The linear surface model developed by regression analysis is

$$F_s = 0.044049+0.0007954* C_C -0.00099553* C_f + 0.021775* Ø_f$$

$$(R^2=0.9995;\ R^2_{adj}=0.9991)$$

**Case: 1** (uncorrelated normally distributed)

The developed performance function

$$G(x) = F_s -1$$

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$X = \frac{x-\mu}{\sigma}$$

X' = matrix of x values

At starting x is taken as mean value of the parameter.

The min distance between the origin and the deign point is reliability index obtained using spared sheet calculation

$\beta = -3.74871$

The failure probability of slope

$P_f = \Phi(-\beta) = 0.999911$

Case2: Correlated normally distributed parameters(c, ø)

The parameters c and ø are linearly correlated with -0.25 coefficient of correlation

C= Correlation matrix

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| | | | |

| | | | |
|---|---|---|---|
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = -3.7657

$P_f = \Phi\ (-\beta) = 0.999917$

The parameters c and ø are linearly correlated with -0.50 coefficient of correlation

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = -3.782956

$P_f = \Phi\ (-\beta) = 0.999923$

Case3: the parameters are un correlated log normal distributed

Min $B_{HL}$ = $\underset{G(Z^*)=0}{min}\sqrt{(X')^t(X')}$

$\beta$ = -3.13191

$P_f = \Phi$ (-$\beta$) =0.99913

Case4: the parameters are correlated log normal distributed

a, coefficient of correlation= -0.25

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

Min $B_{HL}$ = $\underset{G(Z^*)=0}{min}\sqrt{(X')^t(X')}$

$\beta$ = -3.1407

$P_f = \Phi$ (-$\beta$) =0.999157

b, coefficient of correlation= -0.5

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

Min $B_{HL} = \underset{G(Z^*)=0}{min} \sqrt{(X')^t(X')}$

β = -3.1482

$P_f = \Phi\ (-\beta) = 0.999179$

Table: 3.5 Factor of safety (Fs) of dams of 8 sampling points in response surface model by PLAXIS: water table at low level

| | Core | Core | FILL | |
|---|---|---|---|---|
| | $C_C$ | $C_f$ | $Ø_f$ | $F_s$ |
| $\mu$+1.65$\sigma$ | 13.3 | 29.148 | 36.435 | |
| $\mu$-1.65$\sigma$ | 6.7 | 18.852 | 23.565 | |
| | 13.3 | 29.148 | 36.435 | 1.851 |
| | 13.3 | 29.148 | 23.565 | 1.121 |
| | 13.3 | 18.852 | 36.435 | 1.866 |
| | 13.3 | 18.852 | 23.565 | 1.118 |
| | 6.7 | 29.148 | 36.435 | 1.859 |
| | 6.7 | 29.148 | 23.565 | 1.12 |
| | 6.7 | 18.852 | 36.435 | 1.873 |
| | 6.7 | 18.852 | 23.565 | 1.12 |

The linear surface model developed by regression analysis is

$$F_s = -0.21856-0.08661* C_C -0.00063* C_f + 0.057692* Ø_f$$

$$(R^2=0.9998;\ R^2_{adj}=0.9997)$$

**Case: 1** (uncorrelated normally distributed)

The developed performance function

$$G(x) = F_s -1$$

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$X = \frac{x-\mu}{\sigma}$$

$X^{'}$= matrix of x values

At starting x is taken as mean value of the parameter.

The min distance between the Origen and the deign point is reliability index obtained using spared sheet calculation

$\beta = 2.1821$

The failure probability of slope

$P_f = \Phi(-\beta) = 0.014551$

Case2: Correlated normally distributed parameters(c, ø)

The parameters c and ø are linearly correlated with -0.25 coefficient of correlation

C= Correlation matrix

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 2.1821

$P_f = \Phi(-\beta) = 0.014551$

The parameters c and ø are linearly correlated with -0.50 coefficient of correlation

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 2.1821

$P_f = \Phi(-\beta) = 0.014551$

Case3: the parameters are uncorrelated log normal distributed

Min $B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$

$\beta = 2.5137$

$P_f = \Phi(-\beta) = 0.005972$

Case4: the parameters are correlated log normal distributed

a, coefficient of correlation= -0.25

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.25 | -0.25 |
| $C_f$ | -0.25 | 1 | 0 |
| $Ø_f$ | -0.25 | 0 | 1 |

Min $B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$

$\beta = 2.5089$

$P_f = \Phi(-\beta) = 0.006055$

b, coefficient of correlation= -0.5

| | $C_C$ | $C_f$ | $Ø_f$ |
|---|---|---|---|
| $C_C$ | 1 | -0.5 | -0.5 |
| $C_f$ | -0.5 | 1 | 0 |
| $Ø_f$ | -0.5 | 0 | 1 |

Min $B_{HL}$ = $\underset{G(Z^*)=0}{min}\sqrt{(X')^t(X')}$

β = 2.5035

$P_f$ = Φ (-β) =0.006148

From the reliability analysis as per USACE chart we can understand that in dam without any water level both the uncorrelated and correlated variables which are normally or log normally distributed are in the poor to bellow average condition. When the stead state flow is there then the dam is at good condition in the rapid draw down condition dam is going to fail (most hazardous). The steady state flow with low water level is at poor condition.

CHAPTER-4

# RELIABILITY ANALYSIS OF SETTLEMENT OF GEOCELL REINFORCED FOOTING

# 4. RELIABILITY ANALYSIS OF SETTLEMENT OF GEOCELL REINFORCED FOOTING

## 4.1 INTRODUCTION:

In the geotechnical engineering the soil reinforcement concept is being used extensively. The geocell is one of the reinforcement can used in the soil. The settlement of this geocell foundation is analysed by combination of probabilistic and deterministic approach. The limit state function of settlement of geocell foundation on clayey soil for wide range of expected variations in parameters is generated by linear response surface model. The problem is taken from T.G.Sitharam & A. Hegde (2013). The results of experimental were compared with the PLAXIS results. The parameters unit weight(γ) ,angle of shear resistance(ø), young's modulus(E),Poisson's ratio(υ) influences the footing settlement.

## 4.2 FEM Model:

Footing:

Plain strain condition is used to model footing. The footing is considered as strip and placed on the surface of foundation in modelling. The plate element is used to model footing. The modulus of elasticity of steel E=200Gpa is used. The interface is not considered at footing base and soil.

Soft clay:

The behaviour of soft clay is modelled by Elastic- perfectly plastic Mohr Coulomb failure model.

Table: 4.1 clay bed properties

| properties | Mean value |
|---|---|
| Unit weight(γ) | 20.2KN/m$^3$ |
| Young's modulus(E), | 20000Kpa |
| Undrained cohesion(C), | 10Kpa |
| Poisson's ratio(υ) | 0.3 |

Geocell:

The 3-D nature of geocell have not any facility to the user to model in 2D PLAXIS so the geocell in-filled with dry sand was modelled as the composite soil layer with improved strength and stiffness parameters. So many researchers give a report that the geocell in-filled with sand develops apparent cohesion and keeping angle of shear resistance as constant (Rajagopal etal, 1999).

The equations given by rajagopal etal (1999) is used to calculate apparent cohesion

$$C_r = \frac{\sigma_3}{2}\sqrt{K_P}$$

$$\sigma_3 = \frac{2M}{d_0}\left[\frac{1-\sqrt{1-\xi_a}}{1-\xi_a}\right]$$

Where

$C_r$ = increment in apparent cohesion

$\sigma_3$ = confining pressure increment

$K_{P=}$Passive earth pressure

$\xi_a$ = axial strain(2%)

M= secant modulus of geocell material at $\xi_a$ axial strain.

The $C_{r\ value}$ is 50-70% of calculated Cr value from above formula

The Cr value obtained from above equation is 26KN/m$^2$

The friction angle of dry sand consider for experiment is 40$^0$

The dilatancy angle is two third of friction angle.

Mesh generation:

Global coarseness of very fine mesh is generated 15 nodded elements are considered

The figure 4.1 shows the geometrical model of geocell foundation. The figure 4.2 shows the deformed mesh figure 4. 3 shows the vertical displacement

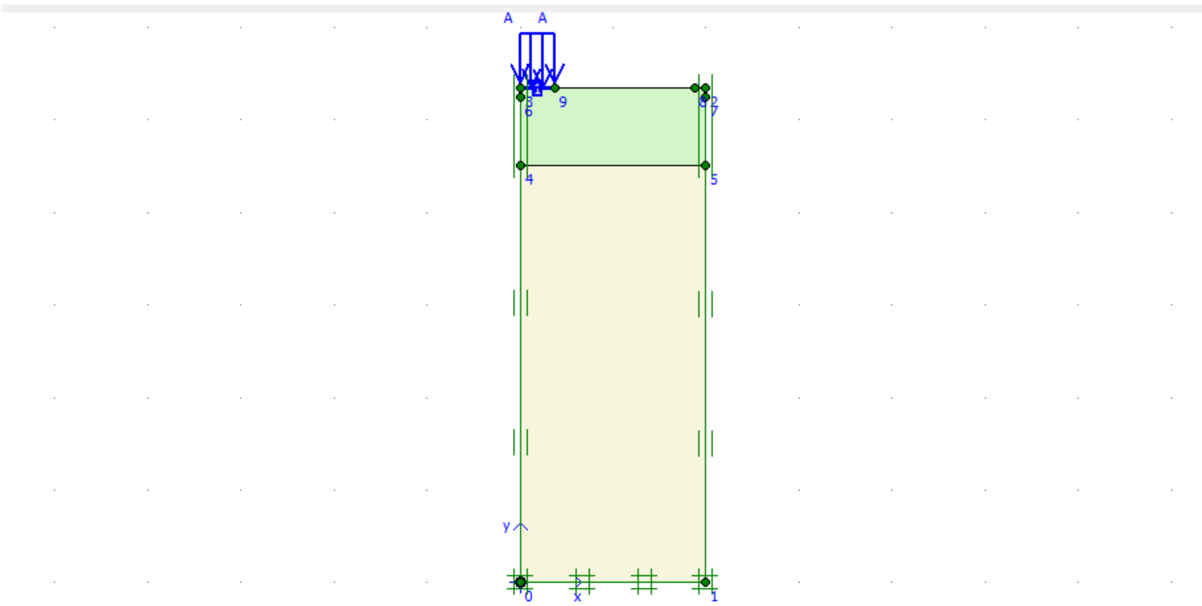

Fig: 4.1 geocell reinforced footing PLAXIS model

## Results:

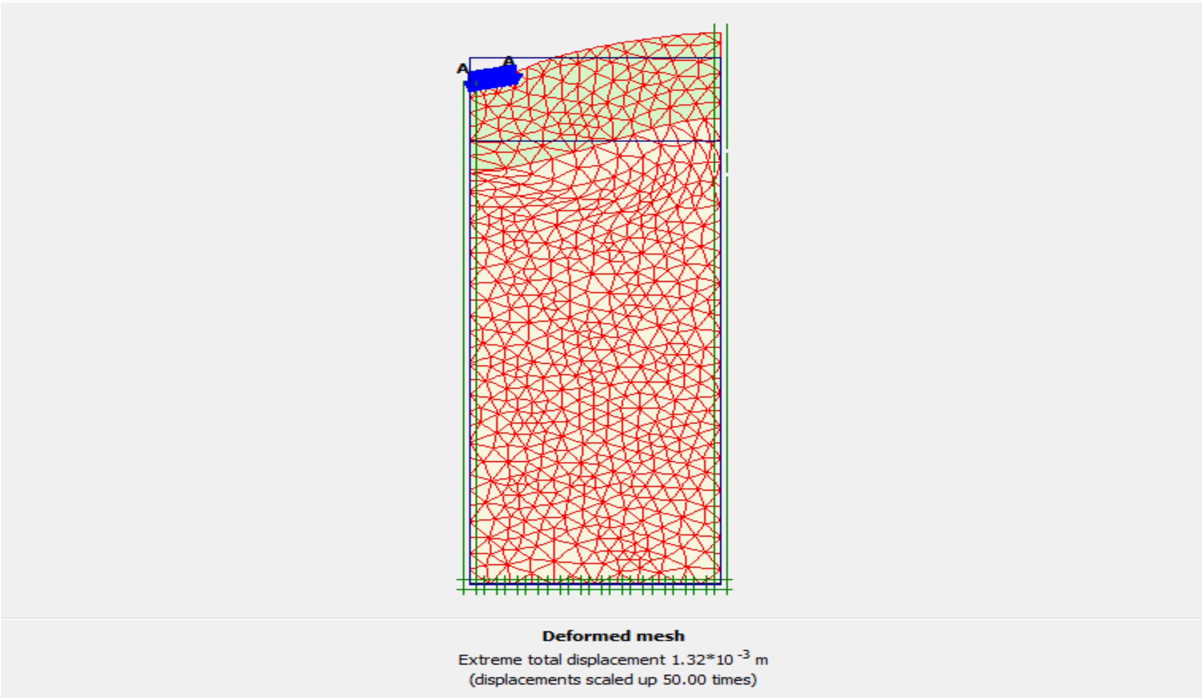


Fig: 4.2 deformed mesh of geocell reinforced footing

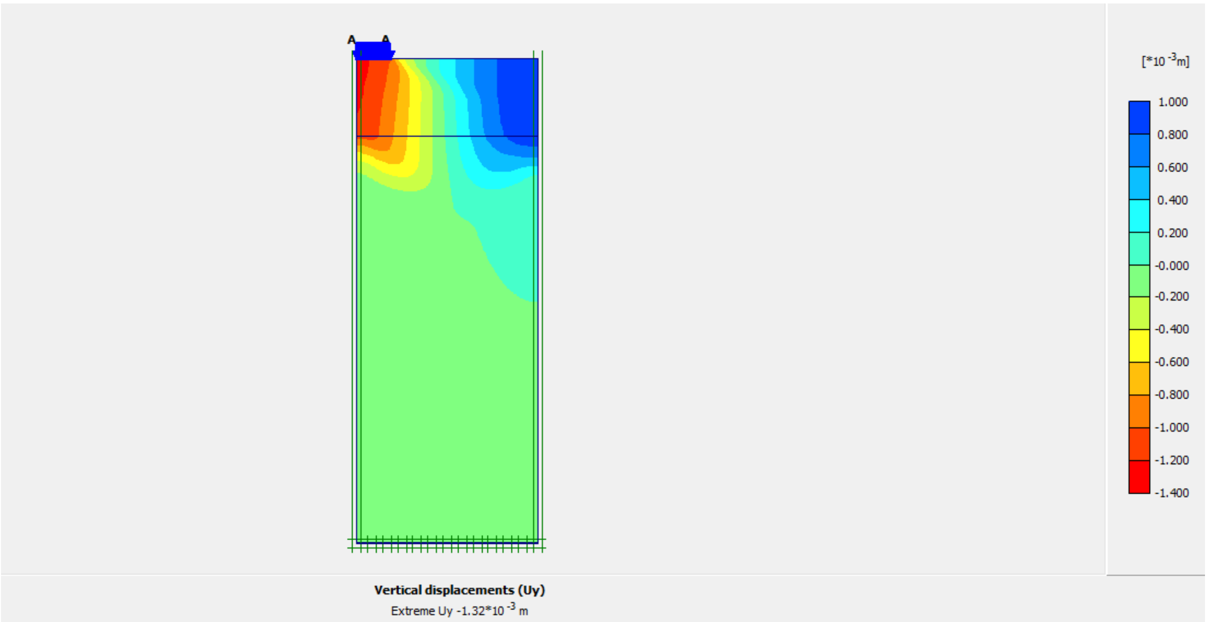


Fig: 4.3 vertical displacement of geocell reinforced footing (relative shadings)

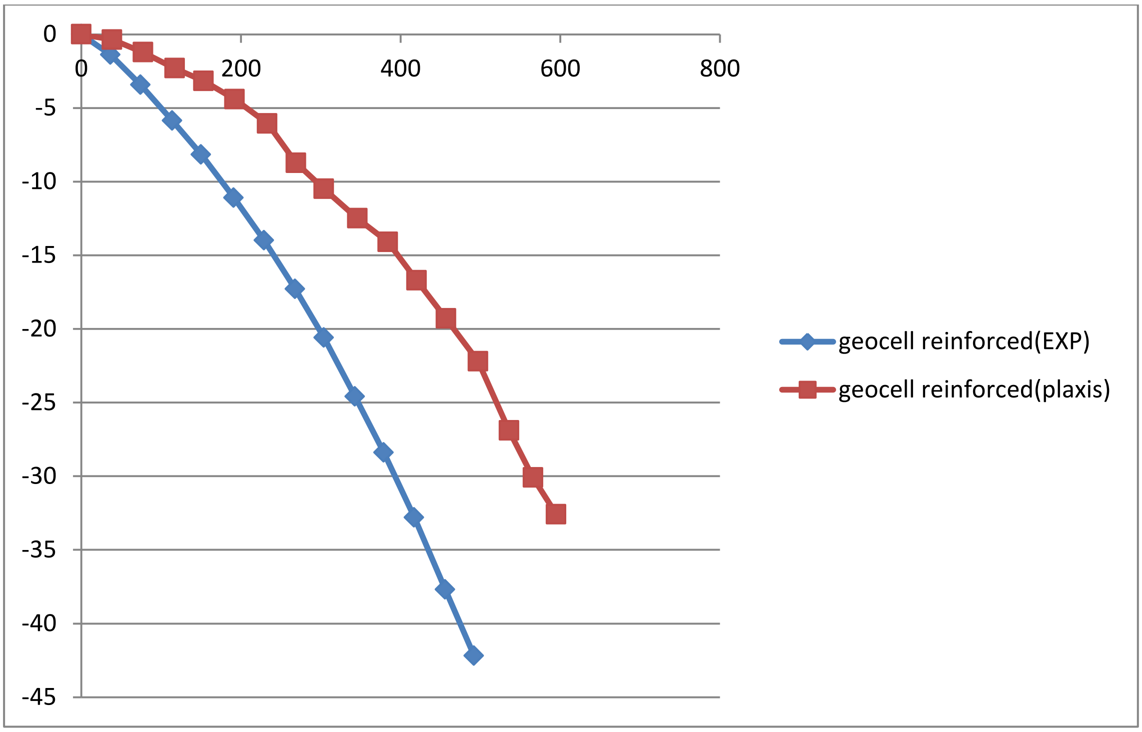


Fig: 4.4 Load-Settlement curve of geocell reinforced footing

**RELIABILITY ANALYSIS:**

Random variation for reliability analysis:

Unit weight (γ), cohesion(c), young's modulus (E) are considered to develop the mathematical model of limit sate function

Table: 4.2 Mean and COV of soil

| | Mean | COV (%) | Standard deviation(σ) |
|---|---|---|---|
| γ(kN/m$^3$) | 20.2 | 7 | 1.414 |
| C(kN/m$^2$) | 10 | 20 | 2 |
| E(kN/m$^2$) | 39000 | 34 | 13260 |

Design of experiments:

Full factorial design model (Mat lab)

```
>>ff2n (3)

    0    0    0

    0    0    1

    0    1    0

    0    1    1

    1    0    0

    1    0    1

    1    1    0

    1    1    1
```

Table: 4.3 Factor of safety (Fs) of footing of 8 sampling points in response surface model by PLAXIS:

| | Γ | C | E | SETTLEMENT(δ)mm |
|---|---|---|---|---|
| μ+1.65σ | 22.53 | 13.3 | 60879 | |
| μ-1.65σ | 17.867 | 6.7 | 17121 | |
| | 22.53 | 13.3 | 60879 | 23.7 |
| | 22.53 | 13.3 | 17121 | 23.6 |
| | 22.53 | 6.7 | 60879 | 41.43 |
| | 22.53 | 6.7 | 17121 | 42.97 |
| | 17.867 | 13.3 | 60879 | 23.72 |
| | 17.867 | 13.3 | 17121 | 23.61 |
| | 17.867 | 6.7 | 60879 | 44.26 |
| | 17.867 | 6.7 | 17121 | 43.07 |

The linear surface model developed by regression analysis is

$$\delta = 65.73616-0.1587*\gamma-2.92045*C-(8E-07)*E$$

$$(R^2=0.996;\ R^2_{adj}=0.993)$$

**Case: 1** (uncorrelated normally distributed)

The developed performance function

$$G(x) = 40-\delta$$

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$X = \frac{x-\mu}{\sigma}$$

$X^{'}$= matrix of x values

At starting x is taken as mean value of the parameter.

The min distance between the origin and the deign point is reliability index obtained using spared sheet calculation

$\beta = 1.273$

The failure probability of footing

$P_f = \Phi(-\beta) = 0.101$

Case2: Correlated normally distributed parameters

The parameters c and ø are linearly correlated with -0.25 coefficient of correlation

C= Correlation matrix

| | γ | C | E |
|---|---|---|---|
| γ | 1 | -0.25 | -0.25 |
| C | -0.25 | 1 | 0 |
| E | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 1.285

$P_f = \Phi(-\beta) = 0.099$

The parameters are linearly correlated with -0.50 coefficient of correlation

| | γ | C | E |
|---|---|---|---|
| Γ | 1 | -0.5 | -0.5 |
| C | -0.5 | 1 | 0 |
| E | -0.5 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 1.29

$P_f = \Phi(-\beta) = 0.09$

Case3: the parameters are uncorrelated log normal distributed

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 1.226

$P_f = \Phi(-\beta) = 0.11$

Case4: the parameters are correlated log normal distributed

a, coefficient of correlation= -0.25

| | γ | C | E |
|---|---|---|---|
| Γ | 1 | -0.25 | -0.25 |
| C | -0.25 | 1 | 0 |
| E | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

β = 1.239

$P_f = \Phi(-\beta) = 0.10$

b, coefficient of correlation= -0.5

| | γ | C | E |
|---|---|---|---|
| γ | 1 | -0.5 | -0.5 |
| C | -0.5 | 1 | 0 |
| E | -0.5 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$\beta = 1.254$

$P_f = \Phi(-\beta) = 0.105$

The results shows the performance of footing is in poor condition according to USACE chart

# CHAPTER-5

# RELIABILITY ANALYSIS OF EMBANKMENT WITH THE STONE COLUMNS

# 5. RELIABILITY ANALYSIS OF EMBANKMENT WITH THE STONE COLUMNS

## 5.1INTRODUCTION:

The embankments placed on the soft ground undergo large horizontal and vertical deformations. The stone columns are one of the ground improvement techniques to strengthen the embankment. In this study probabilistic analysis of stone column embankment was studied. The various strength parameters like cohesion, angle of internal friction and the young's modulus of the embankment material and stone column material influence the stability of embankment during consolidation process. The genetic program is used to develop the mathematical model of limit state function. The $2^k$ design factorial method is used to design the experiments and the FEM package is used for stability analysis. The FORM reliability method is cal the reliability index.

## 5.2 FE MODEL:

The embankment of 5m height with crest width 18m having 1.5H: 1V side slope placed on the 10 m soft clay reinforced with stone column of 0.8m diameter and with a spacing of 2.5m. 30 days is taken to construct each 1m layer and after each layer 30 days is allowed for consolidation the figure 5.21 shows the geometry of the embankment.

Material models:

The sub soil is soft clay is modelled for the Mohr coulomb failure criteria

Table: 5.1 properties of soil (Yogendra (2013))

| properties | Embankment fill | Soft clay | Stone columns |
|---|---|---|---|
| Unit weight($kN/m^2$) | 18 | 15 | 19 |
| Saturated unit weight($kN/m^2$) | 20 | 17 | 20 |
| Yong's modulus($kN/m^2$) | 30000 | 1000 | 35000 |
| Poisson's ratio | 0.3 | 0.35 | 0.3 |
| cohesion($kN/m^2$) | 1 | 5 | 1 |
| Angle of internal frication($^0$) | 30 | 20 | 35 |
| Permeability (m/day) | 1.0368 | $8.64*10^{-5}$ | 10.368 |
| Dilatancy angle | 0 | 5 | 0 |

**Mesh model:**

**Deterministic results:**

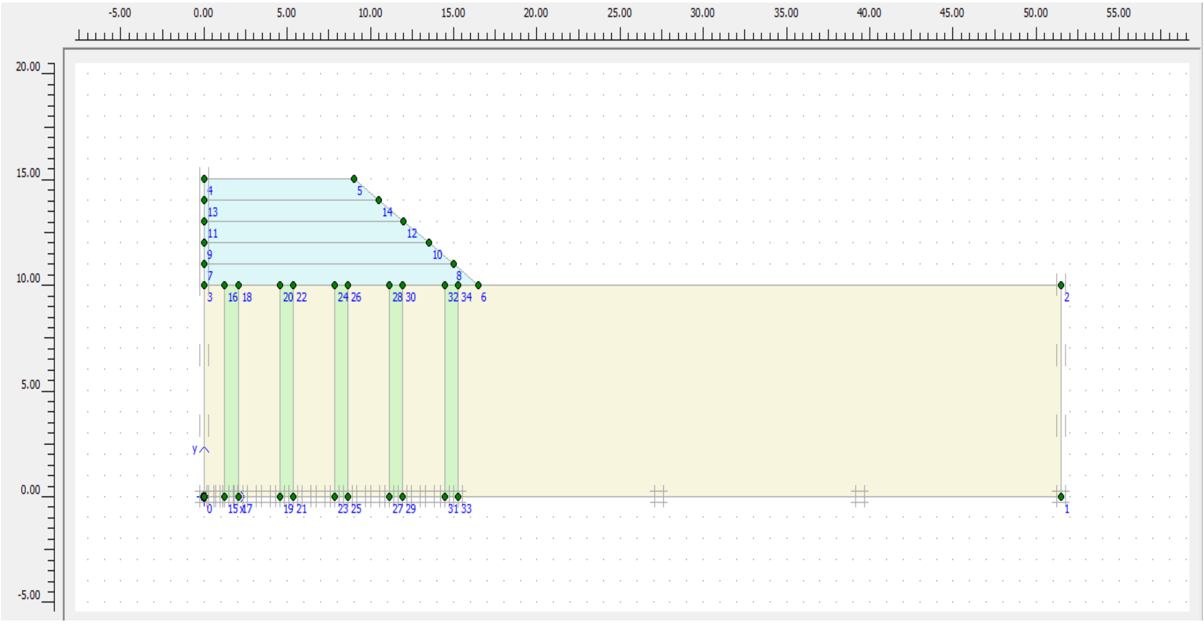

Fig: 5.1 PLAXIS modelling of stone column embankment

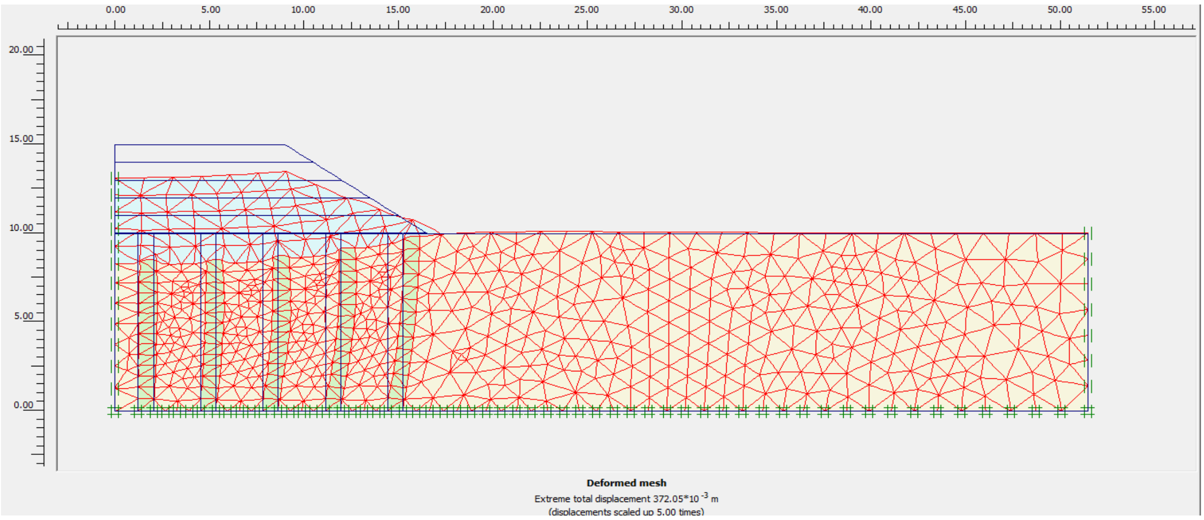


Fig: 5.2 deformed meshes

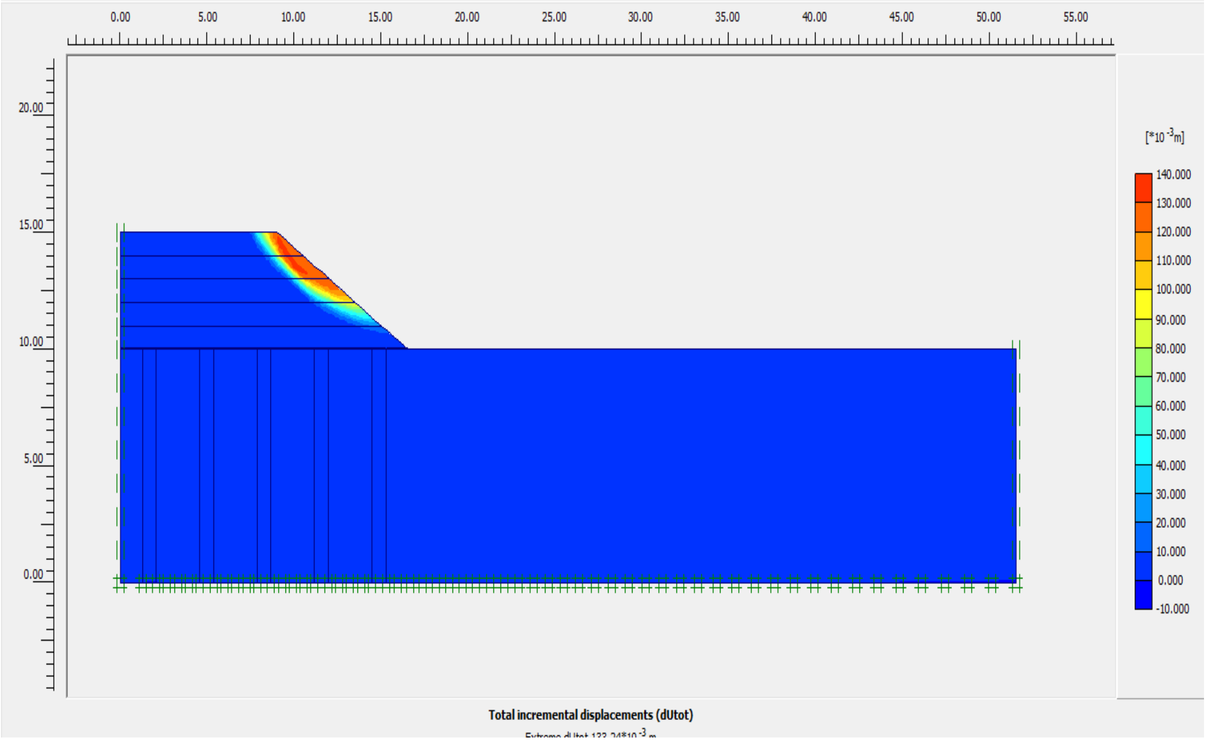


Fig: 5.3 critical failure surfaces

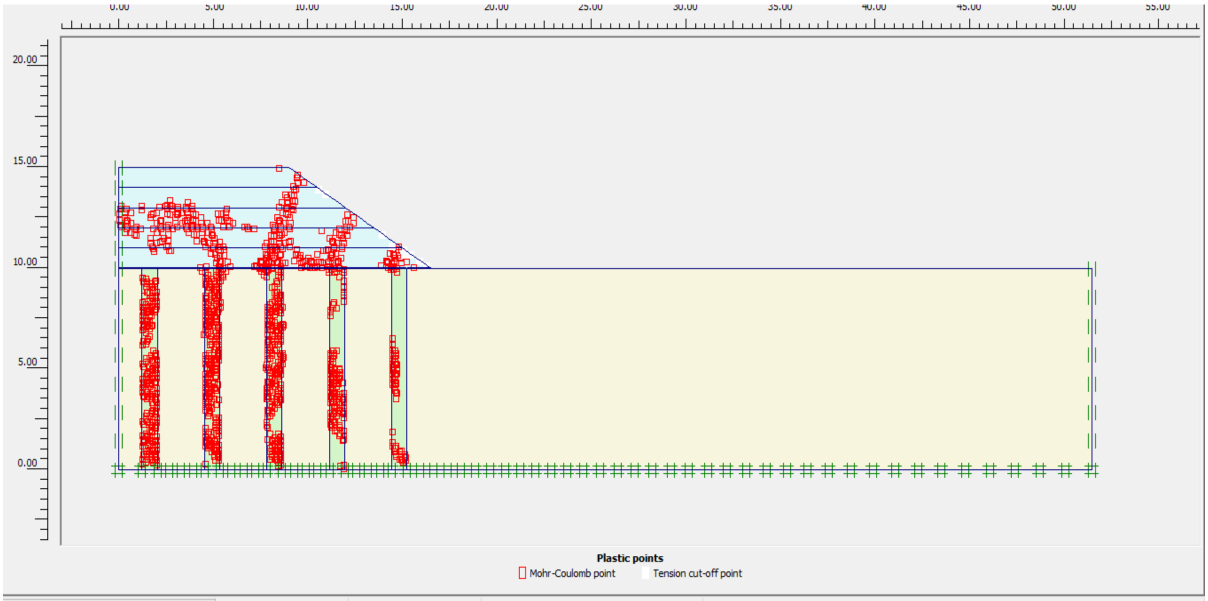


Fig: 5.4 plastic points

## 5.3 RELIABILITY ANALYSIS:

Random variables used in reliability analysis:

Angle of shear resistance (ø) of embankment fill, subsoil, stone column, modulus of elasticity (E) of stone column and cohesion(C) of sub soil to develop limit state mathematical model.

Table: 5.2 Mean and COV of soil

| | COV (%) | Mean | Standard deviation(σ) |
|---|---|---|---|
| $C_{subsoil}$(kN/m$^2$) | 20 | 5 | 1 |
| $Ø_{subsoil}$($^0$) | 13 | 20 | 2.6 |
| $Ø_{subsoil}$($^0$) | 13 | 30 | 3.9 |
| $Ø_{subsoil}$($^0$) | 13 | 35 | 4.55 |
| $E_{stone\ column}$(kN/m$^2$) | 34 | 35000 | 11900 |

Table: 5.3design of experiments for the factor of safety of embankments

| | C | $Ø_1$ | $Ø_2$ | $Ø_3$ | E | $F_s$ |
|---|---|---|---|---|---|---|
| $\mu$+1.65$\sigma$ | 6.65 | 24.29 | 36.435 | 42.5075 | 54635 | |
| $\mu$-1.65$\sigma$ | 3.35 | 15.71 | 23.565 | 27.4955 | 15365 | |
| 1 | 6.65 | 24.29 | 36.435 | 42.5075 | 54635 | 1.343 |
| 2 | 6.65 | 24.29 | 36.435 | 42.5075 | 15365 | 1.343 |
| 3 | 6.65 | 24.29 | 36.435 | 27.4955 | 54635 | 1.34 |
| 4 | 6.65 | 24.29 | 36.435 | 27.4955 | 15365 | 1.346 |
| 5 | 6.65 | 24.29 | 23.565 | 42.5075 | 54635 | 0.883 |
| 6 | 6.65 | 24.29 | 23.565 | 42.5075 | 15365 | 0.883 |
| 7 | 6.65 | 24.29 | 23.565 | 27.4955 | 54635 | 0.887 |
| 8 | 6.65 | 24.29 | 23.565 | 27.4955 | 15365 | 0.888 |
| 9 | 6.65 | 15.71 | 36.435 | 42.5075 | 54635 | 1.261 |
| 10 | 6.65 | 15.71 | 36.435 | 42.5075 | 15365 | 1.306 |
| 11 | 6.65 | 15.71 | 36.435 | 27.4955 | 54635 | 1.244 |
| 12 | 6.65 | 15.71 | 36.435 | 27.4955 | 15365 | 1.306 |
| 13 | 6.65 | 15.71 | 23.565 | 42.5075 | 54635 | 0.858 |
| 14 | 6.65 | 15.71 | 23.565 | 42.5075 | 15365 | 0.851 |
| 15 | 6.65 | 15.71 | 23.565 | 27.4955 | 54635 | 0.866 |
| 16 | 6.65 | 15.71 | 23.565 | 27.4955 | 15365 | 0.884 |
| 17 | 3.35 | 24.29 | 36.435 | 42.5075 | 54635 | 1.379 |
| 18 | 3.35 | 24.29 | 36.435 | 42.5075 | 15365 | 1.345 |
| 19 | 3.35 | 24.29 | 36.435 | 27.4955 | 54635 | 1.261 |
| 20 | 3.35 | 24.29 | 36.435 | 27.4955 | 15365 | 1.282 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | 3.35 | 24.29 | 23.565 | 42.5075 | 54635 | 0.879 |
| 22 | 3.35 | 24.29 | 23.565 | 42.5075 | 15365 | 0.879 |
| 23 | 3.35 | 24.29 | 23.565 | 27.4955 | 54635 | 0.125 |
| 24 | 3.35 | 24.29 | 23.565 | 27.4955 | 15365 | 0.846 |
| 25 | 3.35 | 15.71 | 36.435 | 42.5075 | 54635 | 1.031 |
| 26 | 3.35 | 15.71 | 36.435 | 42.5075 | 15365 | 0.592 |
| 27 | 3.35 | 15.71 | 36.435 | 27.4955 | 54635 | 1.006 |
| 28 | 3.35 | 15.71 | 36.435 | 27.4955 | 15365 | 1.097 |
| 29 | 3.35 | 15.71 | 23.565 | 42.5075 | 54635 | 0.871 |
| 30 | 3.35 | 15.71 | 23.565 | 42.5075 | 15365 | 0.887 |
| 31 | 3.35 | 15.71 | 23.565 | 27.4955 | 54635 | 0.518 |
| 32 | 3.35 | 15.71 | 23.565 | 27.4955 | 15365 | 0.846 |

In this case

The linear surface model developed by genetic programming

$$F_s = 0.004329*X_2-0.0239*X_1+0.0239*X_3-\left(\frac{0.007284*X5}{(X2-X3)^2}\right)-\frac{0.3119*X2*(X1-X3-X5+0.1364)}{X3^2}+\frac{0.0001717*X3*(X2-X3)^2}{X5*(X1-5.521)}-\frac{908*X5}{X1*X3*(X3-X2+X3X4)}+0.2606$$

$(R^2=0.993;\ R^2_{adj}=0.9914)$

**Case: 1** (uncorrelated normally distributed)

The developed performance function

$$G(x) = F_s - 1$$

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$X = \frac{x-\mu}{\sigma}$$

$X'$ = matrix of x values

At starting x is taken as mean value of the parameter.

The min distance between the origin and the deign point is reliability index obtained using spared sheet calculation

$\beta = 0.6199$

The failure probability of slope

$P_f = \Phi(-\beta) = 0.267$

Case2: Correlated normally distributed parameters

The parameters are linearly correlated with -0.25 coefficient of correlation

C= Correlation matrix

| | C | $Ø_1$ | $Ø_2$ | $Ø_3$ | E |
|---|---|---|---|---|---|
| C | 1 | 0 | 0 | -0.25 | -0.25 |
| $Ø_1$ | 0 | 1 | 0 | -0.25 | -0.25 |
| $Ø_2$ | 0 | 0 | 1 | -0.25 | -0.25 |
| $Ø_3$ | -0.25 | -0.25 | -0.25 | 1 | 0 |
| E | -0.25 | -0.25 | -0.25 | 0 | 1 |

$$\text{Min } B_{HL} = \min_{G(Z^*)=0} \sqrt{(X')^t (X')}$$

$$X = \frac{x-\mu}{\sigma}$$

$X'$ = matrix of x values

β = 0.695

$P_f = \Phi(-\beta) = 0.2434$

The parameters are linearly correlated with -0.50 coefficient of correlation

| | C | $\varnothing_1$ | $\varnothing_2$ | $\varnothing_3$ | E |
|---|---|---|---|---|---|
| C | 1 | 0 | 0 | -0.5 | -0.5 |
| $\varnothing_1$ | 0 | 1 | 0 | -0.5 | -0.5 |
| $\varnothing_2$ | 0 | 0 | 1 | -0.5 | -0.5 |
| $\varnothing_3$ | -0.5 | -0.5 | -0.5 | 1 | 0 |
| E | -0.5 | -0.5 | -0.5 | 0 | 1 |

Min $B_{HL}$ = $\underset{G(Z^*)=0}{min}\sqrt{(X')^t(X')}$

$\beta = 0.695$

$P_f = \Phi(-\beta) = 0.2434$

The reliability analysis results shows that the embankment with stone column is under hazardous condition as per USACE CHART because due to the $c_u$<15kN/m$^2$ the stone columns undergoes large lateral displacements.

# CHAPTER-6

# CONCLUSIONS AND SCOPE FOR FURTHER STUDY

## 6.1 Conclusions

In the present study reliability analysis of dam, footing and stone column embankment has been done using first order reliability method (FORM) Hasofer-Lind reliability index and probability of failure was obtained for these cases. Response surface method was used to develop limit state function. The input data obtained from the design of experiments is analysed using Finite Element Method (FEM) PLAXIS 9.02.

In the Chapter-2 basics of Finite Element Method (FEM) in PLAXIS, Response Surface Method (RSM) and Reliability analysis have been discussed. The variables are considered as uncorrelated normally distributed and correlated normally distributed.

In the Chapter-3 stability of dam has been analysed by deterministic method and reliability study was conducted. In the Chapter-4 settlement of geocell reinforced footing was found by FEM method and the probability of exceeding 40 mm settlement was studied. In the Chapter-5 stability of embankment during the stone column was studied.

Based on the present study following conclusions are made.

1. The application of reliability analysis in geotechnical engineering is limited compared to the deterministic methods used. But, considering the uncertainty associated in geotechnical engineering, now reliability analysis is becoming more acceptable.

2. Based on deterministic FEM analysis the dam show the factor of safety is found to 1.57. But based on the reliability analysis the probability of failure is 0.007549 for uncorrelated and 0.007449for correlated for soil parameters (c and ø). This corresponds good in case of steady seepage as per USACE standards.

3. In the absence of in-situ filed data for the geocell reinforced footing, laboratory field data available in literature is considered for comparison of FEM results using deterministic approach. Variability in unit weight (γ), Young's modulus of soil (E), angle of internal friction (ø), interfacial strength between geocell and soil are considered. The results of reliability analysis show the settlement of footing is under poor region as per USACE chart. But it should be noted that in the controlled experiments like laboratory based methods the soil variability is less as compared with the field. This study highlights the importance of reliability analysis in the reinforced footing.

4. In the case of embankment with stone column, the stability of embankment during consolidation stage is analysed by PLAXIS. Taking a theoretical example, due to absence of laboratory or field data. The results of reliability analysis the embankments is at hazards position as per USACE due to large lateral displacement of stone columns due to c(sub soil)<$15kN/m^2$

## 6.2 Scope for further study

Based on the present work it is observed that further intensive study is required in the area of Finite element study and the reliability analysis.

(1) Find non linear limit state function using advanced computation tools

(2) The effect of variation in coefficient of variation of input random variables should be studied.

(3) Reliability study may be conducted for the ADVANCEDFORM methods

(4) Complex geotechnical engineering problems can be solved by reliability system.

(5) In the present study only the settlement of geocell reinforced footing has been evaluated. Reliability analysis can be done by considering the bearing capacity criteria.

# REFERENCES

1. Babu G.L.S and Dasaka, S.M. (2005). Reliability analysis of unsaturated soil slopes. *Geotechnical and Geoenvironmental Engineering, ASCE*, Vol, No.131, No.11, pp.1423-1428.

2. Babu G.L.S, Srivastava A. and Sahana.V (2007) Analysis of stability of earth dams in kachchh region, Gujarat, India. Engineering Geology 94, 123-136.

3. Babu G.L.S and Srivastava A. (2008) Response Surface Methodology (RSM) in the reliability analysis of Geotechnical Systems. 12th IACMAG, Goa, India, 4147-4165.

4. Babu G.L.S, and Srivastava A. (2010) Reliability Analysis of Earth Dams. *J. Geotechnical and Geoenvironmental Engineering.* 136(7), 995-998.

5. Baecher, G. B. (1982) Statistical methods in site characterization. *Updating subsurface samplings of soils and rocks and their in-situ testing*, Engineering Foundation, Santa Barbara, Calif., 463–492.

6. Baecher G.B and Christian J.T. (2003) Reliability and statistics in geotechnical engineering. *John Wiley and Sons.*

7. Basha, B.M and Babu, G.L.S (2008). Target reliability based design optimization of anchored sheet pile walls. *Can. Geotech. J.* 45: 535–548.

8. Bogdanovi, L. (1973). Settlement of stiff structures (silos) founded on soft soil with low penetration resistance. *Transactions*, SR Institute of Testing and Materials, Belgrade, 34.

9. Box, G.E.P., and Wilson, K.B. (1951), On the Experimental Attainment of Optimum Conditions, Journal of the Royal Statistical Society, Series B, 13, 1-45.

10. Bowles, J.E. (2002). Foundation analysis and design. *McGraw-Hill*, New York.

11. Briaud, J. L., and Gibbens, R. (1997). Large scale load test and data base of spread footings on sand. *Rep. No. FHWA/RD-97-068*, Federal Highway Administration, McLean, Va.

12. Brinkgreve R.B.J and Bakker H.L (1991) Non linear Finite Element analysis of safety factors. Computer methods and advances in geomechanics pp 1117-1122. Balkema, Rotterdam.

13. Brinkgreve Ronald B. J. (1999) Beyond 2000 in computational geotechnics: 10 years of PLAXIS International, proceedings of the International Symposium beyond 2000 in Computational Geotechnics, Amsterdam, The Netherlands.

14. Brinkgreve, R.B.J. (2009). PLAXIS 2D-Version (9) *Manual*. Balkema.

15. Burd, H.J. (1999) Beyond 2000 in computational geotechnics: 10 years of PLAXIS International, proceedings of the International Symposium beyond 2000 in Computational Geotechnics, Amsterdam, The Netherlands. pp 35-44.

16. Celestino, T.B., and Duncan, J.M. (1981). Simplified search for noncircular slip surface. *In* Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering, Stockholm, Sweden, pp. 391–394.

17. Chowdhury, R. N., and Grivas, D. A. (1982). Probabilistic model of progressive failure of slopes. *J. Geotech. Engrg. Div.*, 108(6), 803–819.

18. Chowdhury, R. N., and Xu, D. W. (1995). Geotechnical system reliability of slopes.” *Reliab. Eng. Syst. Saf.*, 47, 141–151.

19. Das, B.M. (1999) Principles of foundation engineering, *4th edition, PWS Publishing*, Melbourne,.

20. Fendon G.A. and Griffiths. D.V. (2008). Risk assessment in geotechnical engineering. John Wiley & Sons.

21. Greco, V. R. (1996). Efficient Monte Carlo technique for locating critical slip surface. *J. Geotech. Engrg.*, 122(7), 517–525.

22. Haldar, A and Mahadevan, S (2000). *Probability, Reliability and Statistical Methods in Engineering Design*, John Wiley & Sons, Inc.

23. Harr, M. E. (1987). *Reliability-based design in civil engineering*, McGraw-Hill, New York.

24. Husein et al (2001). An efficient search method for finding the critical circular slip surface using the Monte Carlo technique. *Can. Geotech. J.* 38: 1081–1089.

25. Jaky, J. (1944). The coefficient of earth pressure at rest. “*J. Soc. Hung. Eng. Arch*, 355-358.

26. Low, B. K. (1996). Practical probabilistic approach using spreadsheet. *Uncertainty in the geologic environment: From theory to practice*, C. D. Shakelford, P. P. Nelson, and M. J. S. Roth, eds., ASCE, New York, 1284–1302.

27. Low, B.K., and Tang, W.H. (1997). Efficient reliability evaluation using spreadsheet. *J. Eng. Mech.*, 123(7), 749-752.

28. Low, B. K. (2003). Practical probabilistic slope stability analysis. Proc., Soil and Rock America 2003, 12th Pan-America Conf. on Soil Mechanics and Geotechnical Engineering, and 39th U.S. Rock Mechanics Symp., MIT, Cambridge, Mass., 2777–2784.

29. Low, B.K and Tang, W.H. (2004). Reliability analysis using object-oriented constrained optimization. *structural safety* 26, 69-89.

30. Low, B.K (2005). Reliability based design applied to retaining walls. *Geotechnique,* 55(1), 63-75.

31. Math Work Inc. (2005). *Math Work Inc. Matlab User's Manual*. Version 7.1. Natick (MA).

32. Meyerhof, G.G. (1976). Bearing Capacity and Settlement of Pile Foundations. *J. of Ceo. Eng. Div.* ASCE vol 102: GT3.

33. Murthy, V.N.S. (2003). Geotechnical Engineering: Principles and Practices of Soil Mechanics and Foundation Engineering, *CRC Press*

34. Myers, R.H., and Montgomery, D. (2002). Response surface methodology, wiley, New York.

35. Myers R.H., Montgomery C.D., and Anderson C.M., (2009) Response Surface Methodology – Process and Product Optimization using Designed Experiments Third Edison A John Wiley & Sons, Inc.,Publication

36. Phoon, K.K. and Kulawy, F.H (1999) Evaluation of geotechnical property variability, Can. Geotech. J. 36: 625–639

37. Phoon, K.K (2008). Reliability-based design in geotechnical engineering: computations and applications.

38. Schmertmann, J. H., (1970). Static Cone lo Compute Static Settlement Over Sand, *Journal of the Soil Mechanics and Foundations Division*. ASCE. Vol. 96. No. SM3. Proc. Paper 7302, May, pp. 101l-1043.

39. Tomlinson, M. J. (1991), *Foundation design and construction.* Addison Wesley Longman Limited, Harlow.

40. Subramanian p(2011), Reliability based analysis of slope, foundation and retaining wall using finite element method.

41. Basha b.m (2008), optimum design of retaining structures under static and seismic loading: a reliability based approach.